CHHANDA SHASTRA OF PINGLA - A MATHEMATICAL REVIEW

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ABSTRACT

1) This paper deals with *Chhanda Shastra* a science of meters, invented by Pingalacharya. Panini is considered to be the elder sibling of Pingalacharya (circa 750 BC). Pingalacharya belonged to the village of Shalatur, in the district of Peshawar, now in Pakistan.

2) Definitions
   Prosody – The study of versification
   Mora – The unit to measure the utterance of a syllable. (*Matra*)
   Meter – Rhythmic stucture of a verse.
   Moric Meter – *Matra vrutta*
   Syllabic Meter – *Gana vritta* or *Akshara vrutta*

3) The *matras* have variations:
   Only one variation is short (*Laghu*) denotated by 0.
   A long syllable (*Guru*) denotated by 1.
   There are two variations of two *matras*: 1 or 00.
   There are 3 variations of three *matras*: 01, 10, 11
   There are 5 variation of four *matras*: 11, 010, 001, 100 and 0000 .

In this way, the following table explains the number of the first few *matras* and their respective variations.

No. of Moras.	1	2	3	4	5	6	7	8	9	---
No. of variations. (Moric Meter)	1	2	3	5	8	13	21	34	55	---

This sequence obeys the rule Tn = (Tn-1) + (Tn-2), n>2, n a natural number.

We note that the sequence 1, 2, 3, 5, 8, 13, 21, 34... is a Fibonnaci Sequence

(L. Fibonnaci 1170-1250, Italy).

Pingalacharya states this sequence by the sutra:

'Ankatyopantya Yog Pare Pare Matranam'

Sum of the last two numbers (variations) is that of matra vrutta coming next.

It is interesting to note that the elements of Fibonnaci Sequence are the number of partitions of that number, which reflects Ramanujan`s partition theory. For ex.

4 =

2 + 2	=	11
1 + 1 + 2	=	001
1 + 2 + 1	=	010
2 + 1 + 1	=	100
1 + 1 + 1 + 1	=	0000

(5 Partitions)

4) We can represent any natural number, with base 2, into permutations of 0 and 1 using remainders in successive division, as:

$(14)_{10} = (1110)_2 = (0*2^0 + 1*2^1 + 1*2^2 + 1*2^3 = 14)$

But *Pingalacharya* used the quotients in this division, This method generated a new code as the *Pingala moric binary code* : For ex,14 is represented in 5 bits as

$(14)_{10} = (10110)$,

This can be extended to any number of bits.

5) *Pingalacharya* contributed a moric triangle (*matra meru*) to express a number in short or long bits.

6) Contributions of Pingalacharya:

(i) Discovery of the binary code
(ii) Invention of Fibonnaci Sequence prior to Fibonnaci.
(iii) transformation a given verse into any meter.
(iv) Matra Meru

Pingalacharya was a master of Prosody, the science of patterns of rhythm and sound in poetry, and probably lived around 300 BC. According to Anunduram Borooah (a reknowned Sanskrit scholar of the 19th C), Pingala was not the first to write about prosody but he was amongst the first to study minutely the construction of both syllabic and quantitative meters. Neither did he invent prosody. Poetical meter exists in the earliest known Vedic texts, such as the RigVeda, which may have been composed as far back as 8000 BC. However, attributed to him is the first

text, *Pingalacharya Chhandaśastra* , revealing binary code, the Fibonacci series, the Matra Meru (Pascal's triangle) and the rules for transformation of a verse into any meter.
This paper describes examples of some of his discoveries.

Introduction

Chhanda means meter and Chhandaśastra is the science relating to poetic meter or prosody. Chhanda is one of the six limbs of the Veda which are texts supporting the understanding of those ancient texts.
Pingala introduced combinatorial tools, called *Pratyas*, which can be employed to study the various possible meters in Sanskrit prosody. There is no poetry in Sanskrit without chhanda, or meter.

In order to enjoy and appreciate any poetical composition, knowledge of meter is necessary. Each chhanda has its own mood and flavor. Here, we present a mathematical dimension of *Pingala`s chhanda shastra.*
Pingala belonged to village Shalatur of Peshawar district, now in Pakistan. *Pingala*`s major contribution is mathematical foundation to music (classical, vocal and instrumental) and in particular to versification. Some terms need to be to explained to understand the subject.

1. **Definitions**
   - (i) Prosody = Study of versification
   - (ii) Mora (*matra*) = A unit to measure the utterence of a syllable is called a *matra*. It denotes the time to utter a short vowel. All short vowels are regarded as of one mora (*matra*).
   - (iii) Meter = Rhythmic structure of a verse (*vrutta*).
   - (iv) Moric meter = *Matra vrutta.*
   - (v) Syllable = A vowel with one or more consonants preceding it.
   - (vi) Syllabic meter = *Akshara vrutta* or *gana vrutta.*

Matra is essentially a measure of time. There are two classes of meters in *Sanskrit.*

(i) ***Aksharachhanda* :** There are specified number of syllables they contain. The *vedic aksharachhanda*, in short, is termed as *chhanda* and are specified by the number of syllables. *Aksharachhanda*, also called *vrutta chhanda* consists of 4 lines of a verse, each with a specified sequence of long and short syllables.

(ii) ***Matra chhanda* :** A short syllable is assigned with one matra while a long syllable is assigned two *matras*. In *matra chhanda meters*, the total number of matras is specified.
In *gana chhanda* meters the number of *matras* in each line of a verse is specified.

Pingala algorithms deal only with *vrutta chhanda*. These are of three types: *sama chhanda* (equal), *ardha sama* (half equal) and *visham* (unequal) chhanda.

(iii) A syllable is *laghu* (short) if it has a short vowel. The vowels a, i, u, are short. Even a short syllable will be *guru* (big) if it has a final consonant with an *anuswa*r or a *visarga*.

The combined vowels: aa, ii, uu, ai, au are long vowels. The last syllable of a line of a meter is optioanally taken to be guru.

2. The process of expansion of a *matra vrutta* is similar to that of *varna vrutta.*

A *matra vrutta*, having a single *matra*, has only one variation, namely *laghu* and is denoted by 0.

A *matra vrutta* having a two matras has two variations, one containing guru, denoted by 1 and the other is laghu-laghu, denoted by 00.

The *matras* have variations :

- Only **one** variation is short (*Laghu*) denoted by '0'.
- Two variations of a *matra* is long (*Guru*) denoted by '1'.

There is 1 variation of one *matra* : 1
There are 2 variations of two *matras* : 1 or 00
There are 3 variations of three *matras* : 01, 10, 11
There are 5 variations of four *matras* : 11, 010, 001, 100 and 0000
There are 8 variations of 5 *matras*: 011, 101, 0001, 110, 0010, 0100, 1000 and 00000.
There are 13 variation of 6 *matras* : 111, 0011, 0101, 1001, 00001, 0110, 1010, 00010, 1100, 00100, 01000, 10000, and 000000.
And so on.

Since, 0 counts for one *matra* (*laghu*) and 1 counts for two *matras* (*guru*), it is interesting to note that the sum of *matras* of each variation is the number of that *matra*. The variation 1001 has *matras* 2+1+1+2 = 6, as shown above.
In this way, the following table explains the number of first few *matras* and their respective variations.

No. of Moras.(matra)	1	2	3	4	5	6	7	8	9	---
No. of variations.	1	2	3	5	8	13	21	34	55	---

(Moric Meter)										

This sequence obeys the rule : $Tn = (Tn\text{-}1) + (Tn\text{-}2),\ n > 2,\ n$ a natural number
where *Tn* is *n*th term of a sequence of *matras.*

We note that the sequence *1,2,3,5,8,13,21,34...* is a Fibonacci Sequence by L. Fibonacci (1170-1250, Italy).

It is interesting to note that the elements of variations of matras (and also Fibonacci Sequence) are the number of partitions of that number, which reflects Ramanujan`s partition theory.

For example, a number 4 is partitioned with 5 partitions, which are variations of *matra* 4, as:

2 + 2	=	11
1 + 1 + 2	=	001
1 + 2 + 1	=	010
2 + 1 + 1	=	100
1 + 1 + 1 + 1	=	0000

3. ***Meru Prastar*:** The following is the *Meru Prastara* (triangular extension) of 7 *varnas* in reference to the number of *matra*.

Number of *varna*							number of *matra*
1			1				
1			1	1			$2 = 2^1$
2		1	2	1			$4 = 2^2$
3		1	3	3	1		$8 = 2^3$
4	1	4	6	4	1		$16 = 2^4$
5	1	5	10	10	5	1	$32 = 2^5$
6	1	6	15	20	15	6	1 $64 = 2^6$
7	1	7	21	35	35	21	7 1 $128 = 2^7$

Pingala called this triangle as *varna meru*. It is also known as Pascal`s triangle,
(Blaise Pascal, 1623—1662, France)

4. **Binary representation:**

To convert *14* into *Pingala* binary number consists of 5 *matras*.

Generally, we convert a number 14 (already in base 10), with base 2, and permutations of 0 and 1 using remainders in successive divisions, as:

$(14)_{10} = (1110)_2 = (0.2^0 + 1.2^1 + 1.2^2 + 1.2^3 = 14)$.

It is representation of 14 into 4 *matra* as 1110 contains 4 digits.

But *Pingala* used the quotients in this division, This method generated a new code called the *Pingala moric binary code.*

Since 14 is even, write a *laghu* (0). Now, 14 ÷ 2 = 7, write a *guru* (1). If we get an odd quotient we add 1 to make it even. Hence (7+1) ÷ 2 = 4, hence write 0. Again 4 ÷ 2 = 2, (even), hence write 0. Again 2 ÷ 2 = 1, (odd) hence write guru as 1. In this way, the outcome is (01001) which equals to 14.

That is, 14 is represented in 5 *matras*, as number 01001 containing 5 digits.

5. Permutations

Pingala called permutations as *ankpash*. He posed a question,

In how many ways can the digits of a number 123 be put without repetition?

The answer is: 123, 231, 312, 132, 213, and 321

There are six ways, that is *1 x 2 x 3 = 6* or 3!. Similarly, a 4 digit number, such as 7289, can be expressed in 24 ways that is *1 x 2 x 3 x 4 = 24*, that is 4!.

Number of words : 1596

Reference :

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