The Analysis of Four Fundamental Mathematical operations in Ancient Sanskrit Texts
(In perspective of Addition, Subtraction, Multiplication & Division)

Dr. Daya Shankar Tiwary
Associate Professor
Department of Sanskrit,
University of Delhi, Delhi – 07

Abstract

The basic techniques and principles of Mathematics were so well developed in ancient intellectual tradition that modern scholars are increasingly interested in discovering the knowledge of ancient India. India has a unique distinction of combining the three concepts of the decimal system, place value, and a computational zero (Shunya). We find origin and description of four basic operations in the Vedas, Brahmana Granthas, Ramayana, Mahabharata, Vedangas, Shulbasutras, Aryabhatiyam, Lilavati, Bijaganitam, Brahmasphutasiddhanta, Trishatika, Mahasiddhanta etc. Rigveda (1.164.45), Yajurveda (18.24 and 18.25), Atharvaveda (5.15.1-11), Maitrayani Samhita (1.10.8 and 1.5.8) etc. focus on the root of four basic operations and later mathematicians have explained these functions in their own ways. Four basic operations are also the essential exercise of Vedic Mathematics written by Swami Bharti Krishna Tirthaji Maharaja.

This research paper highlights the analysis of mathematical facts from above mentioned ancient Sanskrit texts for four fundamental mathematical operations. It is the demand of time to co-relate the blend of ancient Indian Mathematics with that of universal modern Mathematics.

Key words: *Vedas, Shulbasutras, Brahmana Grantha, Maitrayani Samhita.*

The *Vedāṅga Jyotiṣa* of Lagadha (1400 BCE) focuses on the significance of Mathematics: –

Yathā śikhā mayūrāṇāṁ nāgānāṁ maṇayo yathā /

Tadvat vedāṅga śāstrāṇāṁ gaṇitaṁ mūrdhani sthitam //

Like the crests on the heads of peacocks, like the gems on the heads of the cobras, Mathematics is at the top of the Vedanga Shastras.

Ancient system of Indian Mathematics may be called Vedic Mathematics. We find references of four fundamental mathematical operations, addition, subtraction, division, and multiplication in the Vedas. The Vedas are the source of whole knowledge system (vedoakhilo dharmamūlam). Vedas are not texts on mathematics but has a mention of lot of mathematical concepts. Vedas provide us with mathematical operations in the form of words or language which are spoken and not in the form of signs (symbols).

Addition

- In *Rigveda* (1.32.14) – *"nava ca navatim ca"* viz. addition of nine and ninety indicates the number nava-navati. i.e. nine ninety. In modern Mathematics, addition can be presented as 9+90 = 99. Here 'ca' means 'and' as a sign word.
- In the mantra of *Rigveda* (4.26.3) "nava sākam navatiḥ", there is addition of nava (9) and navati (90) which is equal to nava-navati (9+90 = 99). Here 'sākam' denotes 'together' which gives the process of 'addition' (+).
- In the *Rigveda* (1.95.1), Agni has two forms first is hari and other is śukra. 2 is analysed as 1+1.
- *Rigveda* 1.164.20, 3.30.11, 1.13.9, 1.164.44 deals with combination of 1+1 and 1+1+1.
- *Rigveda* 7.18.11, 1.45.2, 1.34.11, 10.72.8, 10.72.9 describes the number 21 as an addition of 20+1, 33 as 30+3 or 11+11+11, 107 as 100+7 etc.
- *Taittiriya Samhita* mentions additions of 12 = 10+2 or 6+6, 14 = 7+7, 24 = 6+6+6+6 etc.
- In *Atharvaveda* (19.47.3), we find the series of 99, 88, 77, 66, 55, 44, 33, 22 and 11 (constant difference is - 11).
- Āryabhata-II (910) defines addition, "The making into one several numbers is addition." The name for addition is saṁkalita (made together), saṁkalana (making together), miśraṇa (mixing), sammelana (mingling together), prakṣepaṇa (throwing together), saṁyojana (joining together), ekkikaraṇa (making into one), yukti, yoga (addition) etc.
- Bhāskarācārya-II in his *Līlāvatī* says – "Add the figures in the same places in the direct or in the inverse order."
- Brahmgupta (628) gives us the law of signs of addition – "The sum of two positive numbers is positive, of two negative numbers is negative; of a positive and negative number is their difference."
- Mahāvīra (850), Śripati (1039), Bhāskarācārya-II (1150), Nārāyaṇa (1350) etc also have given laws of sign for addition.

Subtraction

- Rigveda (1.164.45) has a mention of "catvāri vāk parimitā padāni...guhā trīṇi nihitā neṅgayanti turīyaṁ vāco manuṣyā vadanti," in which the remainder 'one' is accounted for when 'three' is subtracted from 'four'.
- In the mantra of *Rigveda* (10.72.8-9), there is reference of subtraction i.e. 8-7 = 1.

- In the mantra of *Atharvaveda* (10.21.3) – "*mā tvāvato jarituḥ kāmam ūnayīḥ,",* root *√ūn* gives the derivative *ūna* meaning 'less than'. The noticeable examples of the word *ūna* signifying subtraction are the number words *eka- ūna-viṁśati* (19), *eka-ūna-trṁśat* (29) etc.
- In *Taittiriya Samhita* (7.4.7) for 49 *"ekasamān-na-pañcāśat"* (= fifty minus one) or even "*ekasyai-na- pañcāśat"* (= one remaining for fifty) is mentioned.
- Āryabhaṭa II (950) has given the definition of subtraction – "The taking out (of some number) from the *sarvadhana* (total) is subtraction; what remains is called *śeṣa* (remainder)". The terms *vyutkalita* (made apart), *vyutkalana* (making apart), *śodhana* (clearing), *pātana* (causing to fall), *viyoga* (separation) etc, have been used for subtraction. The terms *śeṣa* (residue) and *antara* (difference) have been used for the remainder.

Multiplication

- *Rigveda* (8.96.8) refers to the multiplication of $3\times60 = 180$
- In *Yajurveda*, even numbers, Arithmetic progression (AP) i.e. series of multiples of 4 that is also table of 4 [4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 and 48] are mentioned in the following mantra –
- *Catasraśca meaṣṭau ca meaṣṭau ca me dvādaśa ca me ṣoḍaśa ca me ṣoḍaśa ca me viṁśatiśca me viṁśatiśca me caturviṁśatiśca me caturviṁśatiśca meaṣṭāviṁśatiśca meaṣṭāviṁśatiśca me dvātriṁśacca me dvātriṁśacca me ṣaṭtriṁśacca me ṣaṭtriṁśacca me catvāriṁśacca me catvāriṁśacca me catuścatvāriṁśacca me catuścatvāriṁśacca meaṣṭācatvāriṁśacca me yajñena kalpantām//* (Yajurveda – 18.25)
- In *Maitrāyaṇī Saṁhitā* (1.10.8, 1.5.8) $12\times3 = 36,\ 12\times2 = 24$ are mentioned.
- In *Taittirīya Saṁhitā* (7.2.11-20), we find multiples of 2, 4, 5, 10, 20 etc.
- In *Atharvaveda* (5.15.1-11), the multiples of number eleven have been clearly indicated. Terms for multiplication is *guṇana*.
- Brahmagupta (628) mentions four methods: *gomūtrikā*, *khaṇḍa, bheda* and *iṣṭa*. He says: "The product of a positive and a negative (number) is negative; of two negatives is positive multiplied by positive is positive."

The term *abhyāsa* has been used both for addition and multiplication in the *Śulba sūtras* (800 BCE). The multiplicator was termed as *guṇya* and multiplier *guṇaka* or *guṇakara.* The product was called *guṇana-phala* (result of multiplication).

Division

In the operation of Division "*dwidhā, tridhā, caturdhā, pañcadhā* and *anśa"* etc., words are found in the Vedas.

- In *Taittiriya Samhita* (7.1.5), the division of 1000 has been mentioned into three parts.
- Fractions are referred for the first time in *Rig-Veda* (10.90.4). These fractions are called 1/4 *(pada)*, 1/2 *(ardha)*, 3/4 *(tri-pada)* etc.
- *Maitrayani Samhita* (3.7.7). shows the fractions 1/16 *(kala),* 1/12 *(kushtha),* 1/8 *(sapha)* and 1/4 *(pada).* The decimal based system having compound numbers like 11*(Ekadasha-Eka+dasha),* 21*(Ekavimshati-Eka+vimshati),* 27 *(Saptavimshati-Sapta+vimshati)* etc., also indicating the place value system.
- We find the symbolic sign of zero *(shunya)* in *Rigveda* and *"rupe shunyam"* for prastara in *Pingalachhandashastra* (200 B.C.). The idea of expressing all quantities by 1-9 figures and every new series of powers of 10 like *dasha* (2 digit series), shata (3 digit series), sahasra (4 digit series) etc. and *Dvidashati* (*vimshati*=2×10), *panchdashati* (panchashat=5×10) etc. symbolize the concept of zero as integral part of Vedic numeral system.
- *Brihadaranyakopanishad* (2.5.10) describes the number of mysteries of *Indra* as *ananta*.
- Vedic seers were acquainted with the concept of infinity. They were using several words for infinity as *Ananta*, *Pūraṇam*, *Aditi* and *Asaṁkhyāta* etc. *Asaṁkhyāta* is mentioned in the *Yajurveda* (16.54).
- In *Rigveda* (4.35.2-3) "*caturdhā"* has been mentioned as $4\times6 = 24$; 24/4 = 6 or 24/6. Division has been termed as *bhāgahāra, bhājana, haraṇa, chedana* etc. All these terms literally mean "to break into parts," i.e. ""to divide," excepting *haraṇa* which denotes "to take away." This term shows the relation of division to subtraction. The dividend is termed *bhājya, hārya* etc. The divisor *bhājaka, bhāgahara* or simply hara and the quotient *labdhi* "what is obtained" or *labdha*.
- According to Brahmagupta (628) "Positive divided by positive or negative divided by negative becomes positive. But positive divided by negative is negative and negative divided by positive remains negative."

Tribute to Ancient Indian Mathematics

- French mathematician Pierro – Simon Laplace 1749-1827) said "it is India that gave us the ingenious method of expressing all numbers by means of 10 symbols, each symbol receiving a value of position as well as an absolute value. The idea escaped the genius of Archimedes and Apollonius".

- Albert Einstein has marked the Indian contribution “We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made.”
- Indian Scientist Dr. A.P.J. Abdul Kalam has written about the importance of ancient Sanskrit literature - “Ancient Sanskrit literature is a store-house of scientific principles and methodology. The work of our ancient scholars should be thoroughly examined and where possible integrated with modern science”. (Ignited Minds, P.87)

Conclusion

The basic techniques and principles of Mathematics of ancient intellectual tradition are relevant today for the students, researchers and erudite scholars of modern sciences in understanding the origin of Mathematics. This knowledge will bridge the gap between ancient Indian scientific tradition preserved in Sanskrit texts and modern science education at universal level.

References

1. Acharya, Sudumna, 2006, Ganita-Shastra ke Vikas ki Bharatiya Parampara, Motilal Banarasidass, Delhi.
2. Archak, K.B. & Micheal (ed.), 2007, Science, History, Philosophy and Literature in Sanskrit Classics, Sundeep Prakashan, Delhi.
3. Arya, Vedveer, 2014, Indian Contribution to Mathematics & Astronomy, Aryabhata Publications, Hyderabad.
4. Bag, A.K., 1979, Mathematics in Ancient and Medieval India, Chaukhamba Orientalia, Varanasi.
5. Banerjee, M.; Goswami, B. (ed.),1994, Science and Technology in Ancient India, Sanskrit Pustak Bhandar, Calcutta.
6. Bose, D.M; Sen, S.N. Subbarayappa ,B.V.,1971, A Concise History of Science in India, Indian National Science Academy, New Delhi
7. Chaubey, B. B. (ed.), 2009, Rigveda volume I &II, IGNCA, New Delhi.
8. Datta, B.B. 1932, The Science of the Sulba, University of Calcutta, Calcutta.
9. Dwivedi, Sudhakar, 1927, Bijaganitam of Bhaskara II, Banaras Sanskrit series
10. Geetapress (ed.), samvat 2072, Shrimadbhagvadpuran (Vol. I & II), Geetapress, Gorakhapur.
11. Geetapress (ed.), samvat 2045, Mahabharat of Vedvyas, (Vol. I & II), Geetapress, Gorakhapur.
12. Gaur, Ramswaroop Sharma (ed.), 2015, Samveda (Comm. of Shayan), Chaukhamba Publishing House, New Delhi.
13. Jaggi,O.P. 1986, Indian Astronomy and Mathematics, Atma Ram and Sons, Delhi
14. Kashyapa, R.L. (ed.), 2004, Tattirīysamhitā, Sri Aurobindo Kapali Shastri Institute of Vedic culture, Bangaluru.
15. Mishra, K.K. (ed.), 2003, Sanskrit Vānmaya mein Vijñana ka Itihasa, NCERT, New Delhi.

16. Patwardhan, Krishna ji Shankar, Naimpally, Somashekhara Amrita and Singh, Shyam lal (trans.), 2015, Lilavati of Bhaskaracharya, Motilal Banarasidass, Delhi.
17. Prakash, Satya, 1965, Founders of Sciences in Ancient India, The Research Institute of Ancient Scientific Studies; New Delhi.
18. Rao, Balachandra, S., 1994, Indian Mathematics and Astronomy, Jnana Deepa Publications, Banglore.
19. Satavalekara, S.D. (ed), 1942, Maitrayani Samhita, Aundh.
20. Sen,S.N.; Bag, A.K., 1983, Shulbasūtras, Indian National Science Academy, New Delhi.
21. Shānkarbhāshya, Samvat 2061, Brihadāranyakopanishad, Gita Press, Gorakhpur.
22. Sharma, Punita (comm.), 2008, Vedangajyotish,Nag Publication Delhi.
23. Sharma, Ram Swarup (ed.), 1966, Brahmasphutasiddhant of Brahmagupta, Indian Institute of Astronomical and Sanskrit Research, New Delhi.
24. Sharma, Shri Ram (ed.), 2015, 108 Upanishad (Gyankhand)Yuga Nirman Yojana, Mathura, U.P.
25. Sharma, K.V., (ed.), 2009, Brahmand-mahapuran, Chaukhamba Krishnadass Academy, Varanasi.
26. Sharma, Satyadev (comm.), 2008, Aryabhatiyam, Chaukhamba Surbharati Prakashan, Varanasi.
27. Shastri, Ramkrishna (trans.), 1999, Yajurveda (shuklayajurveda), Chawkhambha Vidyabhawan,Varanasi.
28. Tirtha, Bharati Krishna, 2005, Vedic Mathematics, Motilal Banarasi Dass,Delhi.
29. Vasishtha, Shivram Sharma (ed.), 1992, “Shrimadvalmikiramayana”, Chawkhambha vidyabhawan,Varanasi.
30. Whitney, W.D. (trans.), 1996, Atharvaveda, Motilal Banarasi Dass, Delhi.
31. William, M. Monier, 2011, Sanskrit-English Dictionary, Parimal Publications, Delhi.