A Novel Algorithm for Multiplying Four Numbers near Different Bases

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Abstract

Multiplication is one of the most fundamental processes and considered as a laborious operation in mathematics. Vedic maths provides an opportunity to think differently to solve a mathematical problem and makes the process interesting. Currently, there are very few techniques for multiplication of four numbers near different bases.

In this paper, we present a novel approach for performing multiplication of four numbers near different bases. The technique exhibits a special case of multiplication wherein the two numbers are different and may belong to different bases as well. Example, 203*203*305*305 or 506*506*103*103 and so on. The technique makes use of the algebraic equation in solving four numbers near different base which can be considered as a special case for multiplication. Additionally, the technique can be applied in conjunction with doubling & halving technique (sutra) to solve further problems. Different examples will be provided along with proof of the derived approach to facilitate people in learning the new approach.

Introduction

Multiplication is considered as one of the scariest task for students when the numbers participating in the calculation are more than three. There are limited way of performing multiplication using the conventional mathematics techniques. In contrast, Vedic mathematics provides many different methodologies / techniques for performing calculation which are not only faster but easier to remember and implement. Thus, Vedic mathematics has always attracted the attentions of the worldwide researchers towards finding better-optimized techniques for performing the mathematical operations.

The paper is structured as follows: in next section a novel approach for solving four numbers is proposed nearing different bases. The approach will be followed by few examples. Moreover, algebraic proof will be given for the verification of the steps used for solving multiplication of three numbers. It will be followed by the usage of proportional formula to solve different set of numbers. Towards the end, conclusion and future scope will be discussed.

Proposed Methodology

The proposed methodology will contain the following steps for multiplying the four numbers. This represents a special case of multiplication which can be considered as first squaring two numbers and then multiply their result.

Step 1: Represent the four numbers near a different base which needs to be multiplied in the following form

$(x + a)* (x + a) *(y + b) *(y + b)$

where x and y represent the different bases for all the number eg. 200, 500, 1000 and so on, and a, b represent the deficiency or excess near the bases.

For example, if we would like to multiply the numbers 308, 308, 503, 503 these can be represented as follows

$$(300 + 8) * (300 + 8) * (500 + 3) * (500 + 3)$$

Step 2. There are 5 parts to the solution that needs to be calculated as shown in Fig. 1. If there is any carry forward from the previous then it needs to be adjusted properly. Moreover, negative numbers need to be converted before calculating final solution. The number of digits in each part depends on the common base. Common base having values 10, 100 and 1000 will contain 1, 2 and 3 digits in each part of the solution respectively.

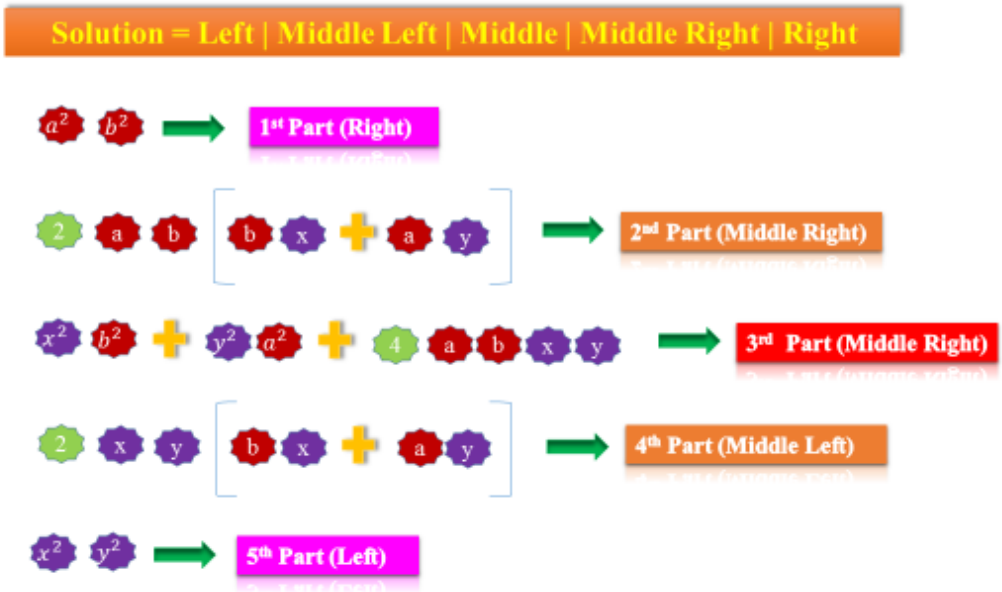

Fig. 1 Stepwise approach to multiply four numbers near different bases.

Example 1

Solve $102 \times 102 \times 305 \times 305$

Step 1) The numbers can be represented in the following form:

$$(100 + 2) \times (100 + 2) \times (300 + 5) \times (300 + 5)$$

2) Calculate the five different parts as mentioned below

1^{st} part = 100

2^{nd} Part = (20*{5+6}) = 220

3^{rd} Part = (25+36+120) = 181

4^{th} Part = (6*{5+6}) = 66

5^{th} Part = 9 The solution is 09|66|181|220|100 = 967832100

Example 2

Solve $98 \times 98 \times 203 \times 203$

Step 1) The numbers can be represented in the following form:

$$(100-2)\times(100-2)\times(200+3)\times(200+3)$$

Step 2) Calculate the five different parts as mentioned below

1st Part = 36

2nd Part = (-12*{3-4}) = 12

3rd Part = (9+16-48) = -23

4th Part = (4*{3-4}) = -04

5th Part = 4

The solution is 4|-04|-23|12|36 = 39577123

Algebraic Proof

$(x+a)(x+a)(y+b)(y+b)$

= $[(x^2+2ax+a^2)(y^2+2by+b^2)$

= $[x^2y^2+2bx^2y+x^2b^2+2axy^2+4abxy+2ab^2x+\ y^2a^2+2a^2by+a^2b^2]$

= $[x^2y^2+2xy(bx+ay)+x^2b^2+4abxy+2ab(by+ax)+\ y^2a^2+a^2b^2]$

= $[x^2y^2+2xy(bx+ay)+x^2b^2+y^2a^2+4abxy+2ab(by+ax)+a^2b^2]$

Using Proportionately

Doubling and halving technique has gained a lot of importance in Vedic mathematics and the sutra can be easily applied to other techniques of calculation. We have extended the algorithm to include numbers which are proportional in nature so that numbers can be first modified based upon the proportionality formula and then after usual process can be followed. Well, that can be best illustrated with the help of a below example.

Example 3

Solve $48 \times 48 \times 804 \times 804$

Step 1) Double 48 to get 96 and half 804 to get 402.

So, the original problem becomes solving $96 \times 96 \times 402 \times 402$

Step 2) The numbers can be represented in the following form:

$$(100-4)\times(100-4)\times(400+2)\times(400+2)$$

Step 3) Calculate the six different parts as mentioned below

1st Part = 64

2nd Part = (-16*{2-16}) = 224

3rd Part = (4+256-128) = 132

4th Part = (8*{2-16}) = -112

5th Part = 16

The solution is 16|-112|132|224|64 = 1489342464

Conclusion and Future Work

In this paper, we have proposed an approach to multiply four numbers which is considered a special case as two numbers are squared and then multiplied to find the answer. It is quite evident from the calculation that the methodology adopted in the paper is fairly straightforward as compared to conventional approach. Additionally, the approach can be extended with proportionality sutra of the Vedic maths to increase the domain space of numbers under multiplication. Further research can be done on the feasibility of leveraging the Vedic way of squaring the number after multiplying the two numbers first.

References

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