A Novel Approach of Multiplying Five Numbers Near a Base

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Abstract

Traditional way of performing multiplication is quite labour intensive and boring as against Vedic technique. Vedic Maths inculcates innovation which gives birth to new techniques for solving a problem. Not only it saves a lot of human effort but also makes the process of solving the problem exciting.

In this paper, we would like to present a novel technique for multiplying five different numbers near a base. The technique makes use of the Pascal triangle and algebraic equation in multiplying five numbers near a base. Moreover, the technique can be blended with other sutras in solving the problem. Proof of the derived methodology and a few examples will be discussed to facilitate the learning the new approach.

Introduction

Conventional method of performing multiplication is quite cumbersome and increases with complexity as the numbers participating in the calculation rises. In contrast, there are various ways to perform multiplication in Vedic Mathematics which are quite easy to remember and implement. Moreover, there are some special cases which can be used to solve multiplication very quickly eg. Multiplication of 2 numbers near 100, 1000 and so on. Squaring of number near a base. There is always a need to have a quite optimized technique in terms of calculating mathematical operations in a quickest way.

The paper is structured as follows: in next section a novel approach for solving five numbers is proposed nearing different bases. The approach will be followed by few examples. Moreover, algebraic proof will be given for the verification of the steps used for solving multiplication of three numbers. After this, usage of proportional formula will be combined to help solve different set of numbers. Towards the end, conclusion and future scope will be discussed.

Proposed Methodology

The proposed methodology will contain the following steps for multiplying numbers.

Step 1: Represent the five numbers near a common base which needs to be multiplied in the following form:

$$(x + a)^* (x + b) ^*(x + c) ^*(x + d)^* (x + e)$$

where *x* represent the common base for all the number, eg. 10, 200, 500, 1000 and so on, and *a*, *b*, *c*, *d*, *e* represent the deficiency or excess near the common base.

For example, if we would like to multiply the numbers 308, 303, 307, 306, 301, these can be represented as follows:

$$(300 + 8) * (300 + 3) * (300 + 7) * (300 + 6) * (300 + 1)$$

Step 2. There are 6 parts to the solution that needs to be calculated as shown in Fig. 1. If there are any carries forward from the previous step then it needs to be adjusted properly. Moreover, negative numbers need to be converted before calculating final solution. The number of digits in each part depends on the common base. Common base having values 10, 100 and 1000 will contain 1, 2 and 3 digits in each part of the solution respectively.

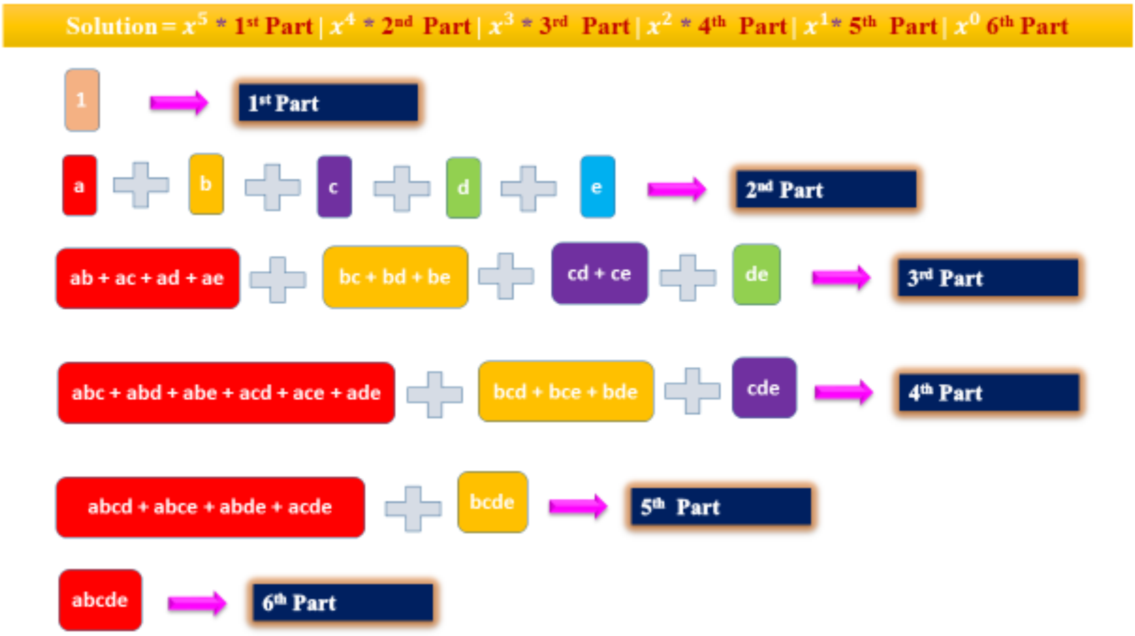

Fig. 1 Methodology to multiply five numbers near a common base

Example 1

Solve $201 \times 203 \times 204 \times 206 \times 205$

Step 1) The numbers can be represented in the following form:

$$(200 + 1) \times (200 + 3) \times (200 + 4) \times (200 + 6) \times (200 + 5)$$

Step 2) Calculate the six different parts as mentioned below

1^{st} part = 1

2^{nd} Part = (1+3+4+6+5) = 19

3^{rd} Part = (3+4+5+6+12+15+18+20+24+30) = 137

4^{th} Part = (12+15+18+20+24+30+ 60+72+90+120) = 461

5^{th} Part = (60+72+90+120+360) = 702

6^{th} Part = 360

The solution is 2^5| 2^4*19| 2^3*137| 2^2*461|2*702|360 = 32|304|1096|1844|1404|360.

After adjusting the carry on digits we get 351514580760 as answer.

Example 2

Solve $102\times99\times98\times104\times105$

Step 1) The numbers can be represented in the following form:

$$(100+2)\times(100-1)\times(100-2)\times(100+4)\times(100+5)$$

Step 2) Calculate the six different parts as mentioned below

1st Part = 1

2nd Part = (2-1-2+4+5) = 8

3rd Part = (-2-4+8+10+2-4-5-8-10+20) = 7

4th Part = (4-8-10-20-16+40+8+10-20-40) = -52

5th Part = (16+20-40-80+40) = -44

6th Part = 80

The solution is 1|08|07|-52|-44|80 = 1|08|06|47|56|80. The answer is 10806475680.

Algebraic Proof

By expanding and collecting like terms it can be shown that

$$(x+a)(x+b)(x+c)(x+d)(x+e)$$
$$= x^5 + x^4\left(a+b+c+d\right)+x^3\left(ab+ac+ad+ae+bc+bd+be+cd+ce+de\right)$$
$$+x^2\left(abc+abd+abe+acd+ace+ade+bcd+bce+bde+cde\right)$$
$$+x\left(abcd+abce+abde+acde+bcde\right)+abcde$$

Using Proportionately

The technique can be further enhanced with the use of proportionality formula of Vedic mathematics. Doubling and halving are the two techniques which comes under proportionality formula. In our technique, there will be additional step to double one number and half the other number under consideration. Well, that can illustrated with the help of an example.

Example 3

Solve $48\times103\times196\times99\times104$

Step 1) Double 48 to get 96 and half 196 to get 98.

So, the original problem becomes solving $96\times103\times98\times99\times104$

Step 2) The numbers can be represented in the following form:

$$(100-4)\times(100+3)\times(100-2)\times(100-1)\times(100+4)$$

Step 3) Calculate the six different parts as mentioned below

1st Part = 1

2nd Part = (-4+3-2-1+4) = 0

3rd Part = (-12+8+4-16-6-3+12+2-8-4) = -23

4th Part = (24+12-48-8+32+16+6-24-12+8) = 6

5th Part = (-24+96+48-32+24) = 112

6th Part = -96

The solution is 1|00|-23|06|112|-96 = 0|99|77|07|11|04. The answer is 9977071104.

Conclusion and Future Work

We have seen that how the proposed approach can be used to solve some of the complex multiplication of five numbers near a base using a simple two step approach. As compared to conventional method of multiplication of five numbers, using the approach, this is quite easy to understand and grab. Moreover, use of doubling and halving formula to make the numbers aligned to common base and then can apply the algorithm successfully. Research needs to be done to allow decimal multiplication of five numbers using extended approach. Moreover, we can exploit other techniques from proportionately apart from doubling and halving to change the numbers under multiplication to near a common base. Another area of research will be to extend the algorithm to get rid of the common base for multiplying five numbers. Lastly, to extend the algorithm for multiplication of higher numbers near a common base.

References

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