CONVERTING A NUMBER TO A PALINDROMIC NUMBER

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What is a Palindromic number?

A Palindromic number is a number that remains the same when its digits are reversed. For example, 121 is palindromic because it is the same whether read from right to left or left to right.

How and where did the idea of Palindromes come from?

The origin of the word comes from the Greek word "palindromos", which means to run back again. *"Palin"* means again and *"dromos"* means way, direction. The first palindromes were made up by organising groups of words and not letters. A Greek poet, Sotades, who lived in Egypt in about (285-246 BC) during the reign of Ptolemy II, wrote a palindrome about the king which wasn't well received. He was sealed in a chest and thrown to sea for his satirical poems.

Can I convert any number to a Palindromic Number?

There is no history about the Palindromic numbers but there were a few people who have tried to find a process to convert numbers to Palindromic Numbers. They used a reverse and add process.

Any number can be converted to a Palindromic number after a stipulated number of iterations. The number of iterations can be predicted based on the number. Originally, to get to a palindromic number, the number was reversed and added to the original number and the process was continued until you got to the palindromic number. For example, the number 23 and reversal of the number gives 32. We add them both i.e. 23+32 to get a palindrome of 55.

In my new approach I have found a method to reach palindromic numbers with an exact number of iterations, this makes the working fun and easy.

Further in this paper you will see how a two-digit number is converted into a palindromic number by using 100, 1000 and 10,000 as a base. It is observed that the number of iterations change depending on the base we choose. Also, if the base is 100, palindromic numbers are 2-digit numbers, if the base is 1000, palindromic numbers are 3-digit numbers, and so on. Towards the end of the paper we will talk about a few exceptions and discuss possible patterns that can be found in the workings.

How do we get to a Palindromic Number?

We start our process keeping in mind the sutra, *All from 9 and the last from 10*. It means that we subtract all the digits from 9 except the unit one's place which is subtracted from 10.

Let us begin by picking a number for example 27.

If 27 is the original number, we need to subtract this from 100 (100 being the base number) to get a complement of 27 which is 73.

The base for 27 is 100 in this example. It could be 1000,10,000 and so on.

First Iteration – 100 - 27 = 73 is the complement number. 27 + 73 = 100.

Second iteration - Reverse the complement 73 (the number we get from the first iteration) to get 37. We arrive at the complement of 37 using the same method (100 – 37 = 63) to get 63.

Third iteration - Reverse the complement 63 to get 36 (using the same method used in second iteration). We arrive at the complement of 36 to get 64.

Fourth Iteration - Reverse the complement 64 to get 46. Then get the complement of 46, 54.

Fifth Iteration - Reverse the complement 54 to get 45. Then the complement of 45 is 55.

First	100 – 27 = 73	27	73
Second	100 - 37 = 63	37	63
Third	100 – 36 = 64	36	64
Fourth	100 - 46 = 54	46	54
Fifth	100 – 45 = 55	45	**55**

So, by using five iterations we have reached the palindromic number for 27 which is number 55.

Can I arrive at a different palindromic number with a different base?

The answer is YES.

Let us keep the original number, which in this case is number 27 and try changing the base to 1000. The complement is then 973.

First Iteration – 1000 - 27 = 973 is the complement number. 27 + 973=1000.

Second iteration - Reverse the complement 973 to get 379.

We arrive at the complement of 379 using the same method (1000 – 379) to get 621.

Third iteration - Reverse the complement 621 to get 126 and take the complement, 874.

Fourth Iteration - Reverse 874 to get 478. Then find the complement of 478 to get 522.

Fifth Iteration - Reverse the complement 522 to get 225, and then find the complement of 225 which is 775.

Sixth Iteration - Reverse the complement 775 to get 577. We arrive at the complement of 577 to get 423.

Seventh Iteration - Reverse the complement 423 to get 324. We arrive at the complement of 324 to get 676.

First	1000 – 027 = 973	27	973
Second	1000 – 379 = 621	379	621
Third	1000 – 126 = 874	126	874
Fourth	1000 – 478 = 522	478	522
Fifth	1000 – 225 = 775	225	775
Sixth	1000 – 577 = 423	577	423
Seventh	1000 – 324 = 676	324	**676**

So, by using seven iterations we have reached the palindromic number for 27 which is number 676.

So, we can see that we need seven iterations to reach the palindromic number for 27 which is number 676 if we change the base from 100 to 1000. Also, we can see that the palindromic number for the number 27 was 55 when the base was 100 and is 676 when the base was 1000. This shows that the palindromic number changes depending on the base number.

Now let us try changing the same number (27) using the base as 10,000.

If 27 is the original number, we need to subtract this from 10000. (10000 being the base number) to get a complement of 27 which is 9973

The base for 27 is 10000 in this example.

First	10000 – 0027 = 9973	27	9973
Second	10000 – 3799 = 6201	3799	6201
Third	10000 – 1026 = 8974	1026	8974
Fourth	10000 – 4798 = 5202	4798	5202
Fifth	10000 – 2025 = 7975	2025	7975
Sixth	10000 – 5797 = 4203	5797	4203
Seventh	10000 – 3024 = 6976	3024	6976
Eighth	10000 – 6796 = 3204	6796	3204
Ninth	10000 – 4023 = 5977	4023	5977
Tenth	10000 – 7795 = 2205	7795	2205
Eleventh	10000 – 5022 = 4978	5022	4978
Twelfth	10000 – 8794 = 1206	8794	1206
Thirteenth	10000 – 6021 = 3979	6021	3979
Fourteenth	10000 – 9793 = 0207	9793	0207
Fifteenth	10000 – 7020 = 2980	7020	2980
Sixteenth	10000 – 0892 = 9108	0892	9108
Seventeenth	10000 – 8019 = 1981	8019	1981
Eighteenth	10000 – 1891 = 8109	1891	8109
Nineteenth	10000 – 9018 = 0982	9018	0982

Twentieth	10000 – 2890 = 7110	2890	7110
Twenty first	10000 – 0117 = 9833	0117	9833
Twenty second	10000 – 3889 = 6111	3889	6111
Twenty third	10000 – 1116 = 8884	1116	8884
Twenty fourth	10000 – 4888 = 5112	4888	5112
Twenty fifth	10000 – 2115 = 7885	2115	7885
Twenty sixth	10000 – 5887 = 4113	5887	4113
Twenty seventh	10000 – 3114 = 6886	3114	**6886**

The palindromic number for 27 with the base 10,000 is 6886, as you can see the number of iterations has increased from 5 to 27. On further calculations it has been observed that the number of iterations does not change even if the base number changes from 10,000 to 100,000 or 10,000,000 for the number 27. Regardless of the change in base number, the number of iterations required to reach the palindromic number remained 27 for the number 27.

Can I determine the number of iterations needed to obtain a Palindromic Number?

The answer is YES.

In the Table 1 below, you can see the palindromic number for the number 6407 has been calculated using the same method for different bases.

TABLE-1

	27		6407	
Base Number	Iterations	Palindromic Number	Iterations	Palindromic Number
100	5	55	NA	NA
1,000	7	676	NA	NA
10,000	27	6,886	71	8,228
100,000	27	68,986	47	61,616
1,000,000	27	689,986	907	649,946
10,000,000	27	6,899,986	407	6,973,796
100,000,000	27	68,999,986	6,407	69,766,796
1,000,000,000	27	689,999,986	6,407	697,696,796
10,000,000,000	27	6,899,999,986	6,407	6,976,996,796

The table below gives an insight to the possible number of maximum iterations for numbers in that base.

TABLE - 2

Maximum Number of Iterations	Base	Numbers
11	100	1 to 99
10	1,000	1 to 999
110	10,000	1 to 9,999
100	100,000	1 to 99,999
1,100	1,000,000	1 to 999,999
1,000	10,000,000	1 to 9,999,999
11,000	100,000,000	1 to 99,999,999
10,000	1,000,000,000	1 to 999,999,999

Conclusion

We have seen that how we can use the proposed approach to obtain a Palindromic Number using different bases. The process becomes cumbersome as the base increases but with the help of computer programming it can be achieved. There were some interesting patterns which were observed.

Firstly, the maximum number of iterations as in Table 2.

Secondly, the sum of consecutive resultant palindromic numbers are multiples of 9 or 11 in one cycle. Cycle meaning numbers having iterations from 1 to 11 in the case of base 100.

Thirdly, I observed an exception to the number 77 (which is already a Palindromic number) when I used it against the base 100 to calculate its palindromic number, the result was the same as the original number, whereas other numbers changed. Could this be as the Digit sum was 5?

This is just the beginning to initiate a research in understanding the process of converting a number to Palindromic number and understanding more patterns.