ETHNOMATHEMATICS: AN EFFECTIVE PEDAGOGICAL TOOL TO ENRICH MATH TEACHING

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Abstract

Ethnomathematics, the term first introduced by the Brazilian educator and mathematician Ubiratan D'Ambrosio, is used to express the relationship between culture and mathematics. The underlying principle of ethnomathematics is recognizing that different modes of thought may lead to different forms of mathematics. Ethnomathematics can be used as an effective teaching tool by teachers not only to enhance the mathematical understandings of students but also to reconstruct the relationship between culture and mathematics. The possibility for alternatives allows students to appreciate mathematical ideas from different cultures across different time periods and gives them a better perspective of the historical and scientific evolution of mathematics. Ethnomathematics encourages students to learn to appreciate the achievements of their own and other cultures.

Ethnomathematics

The term "ethnomathematics" was introduced by the Brazilian educator and mathematician Ubiratan D'Ambrosio in 1977 during a presentation for the American Association for the Advancement of Science. Ethnomathematics is used to express the relationship between culture and mathematics. The mission of ethnomathematics is to acknowledge that there are different ways of doing mathematics by considering the appropriation of the academic mathematical knowledge developed by different sectors of the society as well as by considering different modes in which different cultures negotiate their mathematical practices. (D’Ambrosio, 1993)

Quite early in their schooling, most students learn to hate math or believe that they cannot “do” math, as it is defined by the traditional academic approach (Mukhopadhay, Greer & Roth, 2012). Math here is school math or a Eurocentric way of knowing math. Based mainly on Greek texts together with developments in North West Europe in the 17th and 18th centuries, Eurocentric math has become the de facto standard way of understanding the world of math (Ascher & D’Ambrosio, 1994). Currently, most histories of mathematics are almost entirely Eurocentric (G.Joseph)

To embrace a broader idea of what constitutes mathematics, like language and social studies, the study of mathematics should aim to engage ideas from different cultures across the whole world. One way to do this is to include aspects of ethnomathematics in math teaching to help students understand the mathematics that exist beyond the framework of the school math. (Brandt & Chernoff)

This study is focused on some ethnomathematical or non-traditional algorithms that teachers can use in their classroom to not only enhance the mathematical understandings of students but also to reconstruct the relationship between culture and mathematics

Finger Counting

Finger-counting, also known as dactylonomy, is the act of counting using one's fingers. The techniques vary across different geographical areas, ethnicities, and time periods. In Japan people count from 1 with the index finger, French and German people begin with the thumb, while in Philippines they begin with little finger. In India, people use the joints of the index finger or the little finger to represent 1. Across the world there are 27 types of counting method using the fingers. (Nishiyama, 2013).

The degree of cultural diversity in finger counting, however, has been grossly underestimated in the field at large [Bender & Beller, 2012]. In their paper, Andrea Bender and Sieghard Beller demonstrate that fingers as a tool for counting are not only naturally available but are also—and crucially so—culturally encoded.

Citing this research, Corrinne Burns in her blog in the Guardian raises some extremely pertinent questions like: Knowing that there is a link between hands and numbers, and that how we process numbers mentally is influenced by how we finger-count, what are the implications of the vast cultural diversity in techniques? Does it mean that we think about numbers differently, depending on our cultural background? She concludes that, if this is true, then the study and understanding of finger counting methods can be a useful tool for the teachers in multicultural classrooms, where students bring their own cultural learning to the classroom. Exploring finger counting techniques from different parts of the world, with deeper explorations/investigations into the history and social background can help students to make cross cultural connections and foster respect for the cultures around the world.

Fig.1 shows a pictorial representation of Finger Counting in some parts of the world

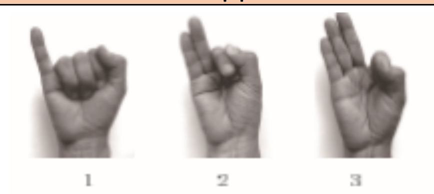
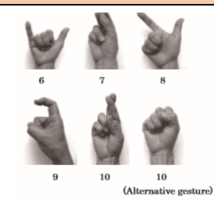
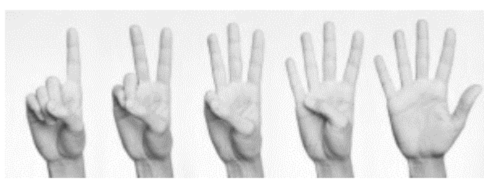
Counting with the thumb Germany and France	**Bending the fingers beginning with the thumb** Japan and China (1 to 5)
	
Counting with the little finger Philippines	**6 to 10 on one hand** China
	
Counting with the index finger US and UK	**Counting using the joints of the fingers** India, Pakistan and Bangladesh
	

Figure 1

Multiplication Using Fingers

The use of fingers can be extended from counting on hands to computing products of simple numbers. The most popular algorithm is the 17th century French peasant or Cajun Multiplication. The algorithm was used/developed to compute the products of numbers from 6 through 10.

Two hands are used in the algorithm, each hand representing a factor in the product. Finger positions shown are as shown below.

6 7 8 9 10

Figure 2

Example Calculate: 8 x 7 (Using Cajun Multiplication)

The finger representation of the two numbers will be as shown below:

8x7

Figure 3

To find the product,

Calculate the sum of the raised fingers (3 + 2 = 5). This represents the number of tens.

The product of the closed fingers is the number of ones (3 x 2 = 6)

$$8 \text{ x } 7 = (3 + 2) \text{ x } 10 + 6 = 56$$

Algebraic Proof of the Algorithm

a x b = [(a - 5) + (b – 5)] x 10 + [(10 – a) x (10 – b)

= [a – 10 + b] x 10 + 100 – 10b – 10a + ab

= 10a – 100 + 10b + 100 – 10b – 10a + ab ----------------- Step A

= ab

Dr. Bethany Noblitt and Blaire Richter of Northern Kentucky University in their paper, *Using Vedic Mathematics to Make Sense of the Finger Algorithm*, drew resemblances between the Cajun Multiplication using Finger algorithm and finding products of two numbers using the Nikhilam Sutra of Vedic Mathematics. Vedic Mathematics is a system from India, first written by a spiritual teacher named Bharati Krishna Tirtha (1884 – 1960). Fired up by his own love of mathematics he followed various clues in the Vedas and came up with a system based on sixteen rules or sutras. (Tirthaji B.K. 1965).

Calculate 8 x 7 (Using Nikhilam Sutra)

	LHS	RHS	
	8	2	**(10-8)**
X	7	3	**(10-7)**
(8-3) or (7-2)	**5**	**6**	**(2X3)**

Algebraic Proof of Nikhilam Multiplication a x b

	LHS	RHS
	a	(10-a)
x	b	(10-b)
	[a-(10-b)] or b-(10-a)	[(10-a) x (10-b)]

a x b = [a-(10-b)] x 10 + [(10-a) x (10-b)]

= 10a – 100 + 10b + 100 – 10b - 10a + ab--------Step A

= ab

The algebraic expressions of Step A from the algebraic proof of finger multiplication and Nikhilam multiplication are the same.

Dr. Bethany and Blaire point out that this relationship is significant because it gives the teachers a perfect opportunity to explore and justify two nonstandard multiplication algorithms, while making mathematical connections between them, and providing them with exposure to a culturally different way of viewing mathematics. They advocate the use of these different non-standard algorithms as intermediate and temporary procedures that serve as a bridge between first exposure to arithmetic calculations and quick recall of basic math facts.

Math, Culture and beyond

More inquisitive minds can be encouraged to investigate ways of multiplying numbers beyond 10 using Vedic Mathematics and finger algorithms. This helps improve their math skills and develop a better understanding of numbers. Teachers can also help students appreciate knowledge from different cultures by guiding them to the study of Vedas.

Mathematical advantages of using fingers

Canadian researchers, Marcie Penner-Wilger and Michael L. Anderson, propose that the part of our brain that originally evolved to represent our fingers has been recruited to represent our concept of numbers, and that these days it performs both functions. MRI scans show that brain regions associated with finger sense are activated when we perform numerical tasks, even if we don't use our fingers to help us complete those tasks. The study, *Does finger training increase* *young children's numerical performance?* by Gracia-Bafalluy M and Noël MP shows that young children with good finger awareness are better at performing quantitative tasks than those with less finger sense.

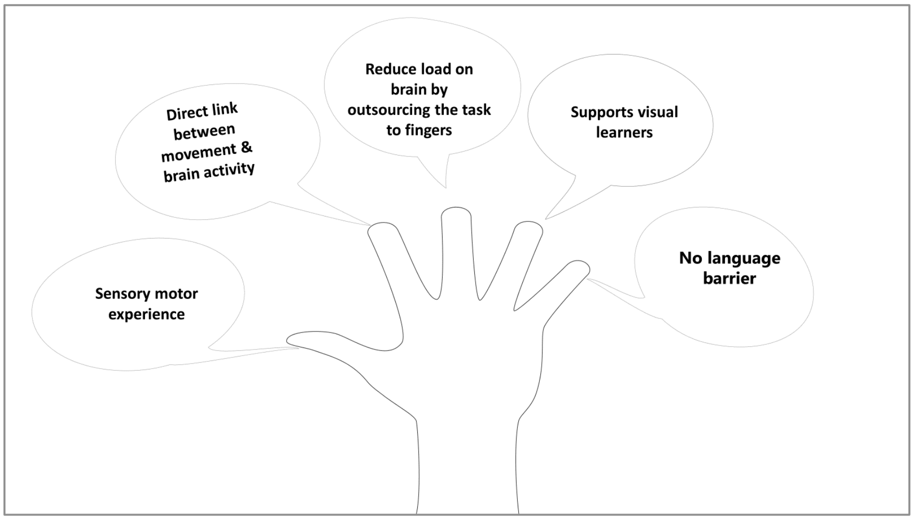


Figure 4

Figure Tracings and Graph Theory

We will understand figure tracings in four different cultures across the world to gain better insights into the cultural context of each, and then see how this can help teachers introduce the concept of graph theory in the classroom. Figure tracings follow the specification that each line be traced one and only once without lifting your finger from the ground.

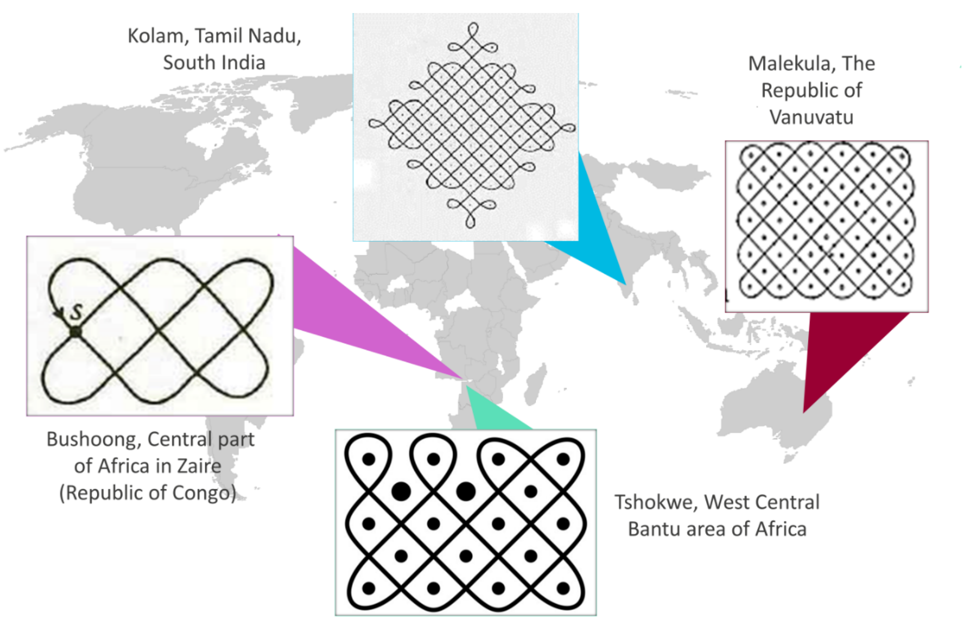


Figure 5

Bushoong, Central part of Africa in the Democratic Republic of Congo

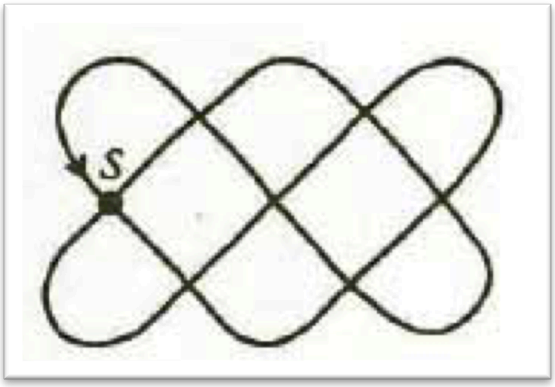


Figure 6

- Children play games drawing figures in the sand
- Each figure has a distinct name, which depends on how it is categorized
- The Bushoong view a design as composed of different elementary designs, and the name given to a figure is the name associated with its most significant constituent
- Significance is also related to the process, and so sculptors may differ from embroiders in the names they assign
- The drawings are a continuous path with a start and an end point
- Understanding this concept is intrinsic to children.
- There is no record of the tracing path they used

Malekula, Republic of Vanuatu, Group of Pacific islands

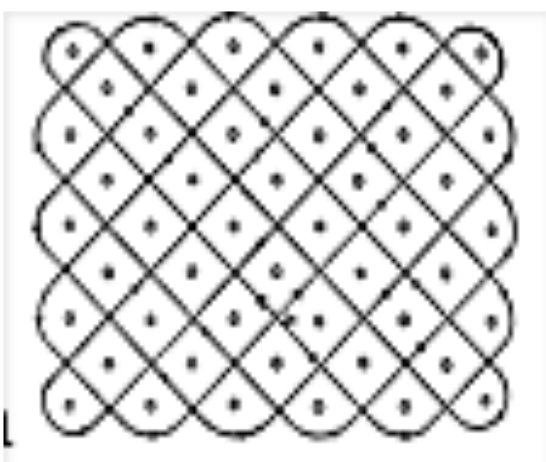


Figure 7

- The drawings (called nitus) are drawn by men in sand and the knowledge is handed from generation to generation
- Drawing of continuous figures is tightly enmeshed in the Malekulan ethos
- The figure is drawn with a single continuous line, the finger never stopping or never lifted from the ground, and no part covered twice
- Many figures are named for flora and fauna, but several are related to myths and rituals
- In one of the pattern tracing begins with some initial pattern which is repeated within or around itself getting bigger and bigger, or smaller and smaller.

Tshokwe. West Central Bantu area of Africa

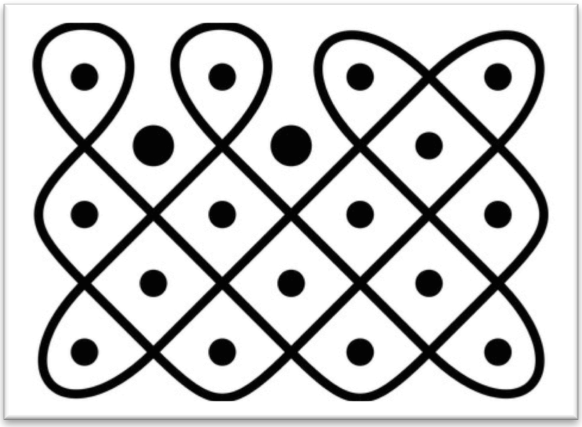


Figure 8

- In Tshokwe, drawing continuous figures in the sand is a part of a widespread storytelling tradition
- The figures are called Sona and are drawn exclusively by men
- The skill combines the memory of the drawings, the flowing movement of the fingers through the sand, and the added art of storytelling
- The most crucial element of all stories is a simple closed planar curve which determines two regions of which it is a common boundary. This is the Jordan curve theorem: any continuous simple closed curve in the plane, separates the plane into two disjoint regions, the inside and the outside.

Kolam, Tamil Nadu, South India

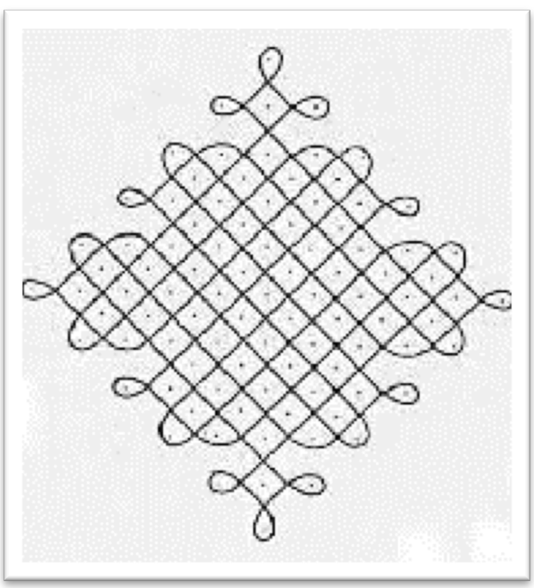


Figure 9

- Kolams are curved line patterns drawn by women every morning in front of their houses after sprinkling water and cleaning the ground in South India.
- Traditionally this is done using rice flour and is not intended to be a permanent design.
- This ritual of creating patterns on the floor (at the entrance of a household) has been passed down from generations over thousands of years.
- Millions of Tamil women still practice this ritual and it is embedded in their everyday life.
- This art form is also practiced in other parts of India with different names
- Kolam skills are viewed as a mark of grace and as a demonstration of dexterity, mental discipline and ability to concentrate

Graph Theory

The history of graph theory can be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island—but without crossing any bridge twice. Euler argued that no such path exists.

Euler experimented to draw paths first in the graph that he drew (see the pic below), the first known visual representation of a modern graph. He eventually extrapolated a general rule that such a path can exist only if all vertices in the graph have an even degree by later experimenting with multiple theoretical graphs with alternating number of vertices and edges.

Between Euler in 1736 and Heirholzer some 130 years later, graph theory was established.

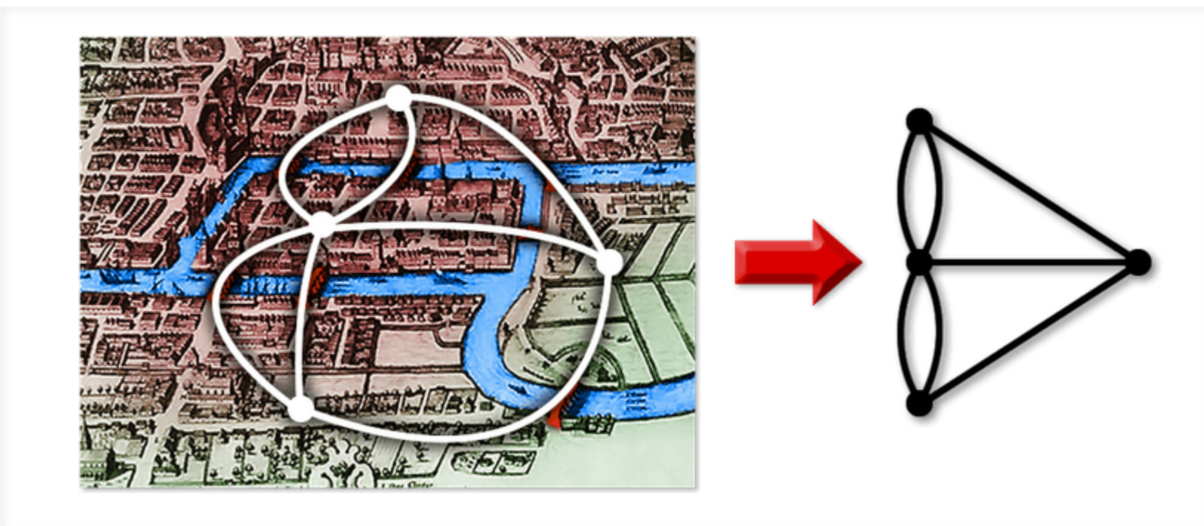
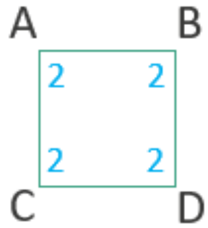
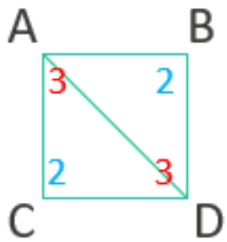


Figure 10

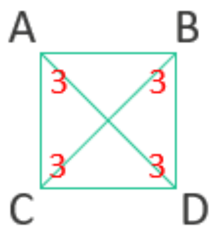
According to the theory, a loop can be completed only when the number of edges (degree of a vertex) meeting at all the vertices are even, or when only one pair of odd edges meet at the vertices. When meeting at all the vertices is even, the path can start from any vertex and end where it began. This is called as Eulerian circuit. When only one pair of odd edges meet at the vertices, the path starts at one odd vertex and ends at the other. This is called a Eulerian path. In both the cases there can be more than one way to complete the loop.



Edges: AB, BC, BD, DC
Vertices: A, B, C, D
Degree of Vertices: A – 2, B – 2, C – 2, and D – 2
ABCD forms the Eulerian circuit with a start at A and end at A.
Other possible paths for the Eulerian circuit: BDCA, DCAB, and CABD



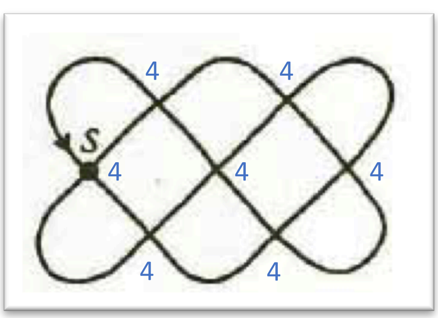
Edges: AB, BC, BD, DC
Vertices: A, B, C, D
Degree of Vertices: A – 3, B – 2, C – 2, and D – 3
ABDACD forms the Eulerian path with a start at A and end at D.
Other possible paths for the Eulerian path: DBADCA, DCADBA, ACDABD (Always starts at one odd vertex and ends at the other odd vertex)



Edges: AB, BC, BD, DC
Vertices: A, B, C, D
Degree of Vertices: A – 3, B – 3, C – 3, and D – 3
Eulerian path or circuit is not possible, since the degrees of all the vertices are odd.

Making Connections

With this new understanding of graph theory let us look again at the figures that were traced by people in different parts of the world. The fundamental question in graph theory is, for a connected planar graph, can a continuous path be found that covers each edge once and only once?



The sand drawing made by the children of Bushoong

This is an example of Eulerian path. The degree of all the vertices is 4.

Similar patterns can be observed with other figures discussed above. It is possible to draw Eulerian path or the Eulerian circuit on these strikingly beautiful drawings, unique to each culture.

This critical insight leads us to the draw the connection between the games played by the children (Bushoong), stories told by the seniors (Sona), rituals by women (kolams), or Sunday strolls in Konigsberg. Teachers can start by directing attention to the difference in culture, and differences in the context of these figures and then build the lesson around the shared understanding of the mathematical concept of graph theory.

Conclusion

Ethnomathematics lends itself naturally to an interdisciplinary approach to curriculum integration. It can help generate a deeper understanding of mathematical themes and concepts that cut across different disciplines and their relationships to the real world. It fits well within the constructivist theory of having students build understanding and knowledge through what they have already learned and been exposed to previously (Brandt & Chernoff). Students feel valued for the unique experiences and perspective they bring and develop the key life value of listening and learning from each other. Although there are wrinkles that need to be ironed out (i.e., worries of turning this into folklore or romanticizing foreign cultures), these can easily be addressed by educators who think critically and creatively about the material they share with the students and how it is presented. (Brandt & Chernoff).

To conclude, Ethnomathematics can be a useful tool to help students succeed in mathematics, because success is more linked to opportunities to learn in a meaningful way than to innate intelligence. Giving each student their voice and choice in learning can make all the difference.

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