INDISPENSABILITY OF NUMBERS AND NUMERALS OF INDIAN INTELLECTUAL TRADITIONS AND THEIR SCIENTIFIC ROLE

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Abstract

The Science of Mathematics with all its branches such as Arithmetic, Algebra, Geometry and Trigonometry etc. was so well developed in ancient India that modern scholars are increasingly interested in discovering the knowledge of ancient India with its unique distinction of combining the three concepts of the decimal system, place value and a computational Zero (Śūnya).The first literary evidence of Mathematics is mentioned in the Vedas. We find clear description of numeral system, decimal system, place value and Zero in the Vedas, Brāhmaṇa Granthas and the magnum opus - the Rāmāyaṇa and the Mahābhārata, Vedāñgās, Śulbasutras, Âryabhaṭīyam, Līlāvatī, Bījagaṇitam etc. In the Yajurveda (17.2), the powers of 10 from $10^{º}$ to 10^{12} is listed. Taittirīya Saṁhitā (7.2.11-20) also mentions this list of numerals. we also find the series of Arithmetic Progressions and Geometric Progressions. The research paper is aimed at determining the mathematical facts ,with examples and proofs, from ancient Sanskrit texts as mentioned above with scientific approach.

Key words : Vedāṅga Jyotiṣa, Vedic Mathematics, Yuga, Kuṭṭaka.

The importance of mathematics has been highlighted in the *Vedāṅga Jyotiṣa* (1400 BCE) of Lagadha :

Yathā śikhā mayūrāṇāṁ nāgānāṁ maṇayo yathā /
Tadvat vedāṅga śāstrāṇāṁ gaṇitaṁ mūrdhani sthitam //
(Vedāṅga Jyotiṣa - 4)

i.e. Like the crests on the heads of peacocks, like the gems on the heads of the cobras, Mathematics is at the top of the Vedanga Shastras. Ancient system of Indian Mathematics may be called Vedic Mathematics. Jagadguru Shankaracharya Shri Bharati Krishna Tirtha ji (1884-1960) had formulated sixteen Sutras and their thirteen corollaries (sub-sutras) on the basis of the Vedas. The whole text is known as "Vedic Mathematics".

Description of the Numeral System, Decimal System, Place Value, Zero and Infinity

We find clear description of Numeral System, Decimal System, Place Value and Zero in our Vedas, Brāhmaṇa Granthas and the magnum opus- the *Rāmāyaṇa* (6th BCE) and the *Mahābhārata* (4th BCE) and other texts. The Vedic names of numerals being used today indicates that the decimal system assigning symbols 1-9 and the concept of place value and zero were fully evolved during Vedic period. There is clear reference of numbers in the Vedas. In the *Ṛgveda* **(**1500-1000 BCE**),** we find one (*Ekam*) and twenty (*VinŚati*) numbers in this mantra –

Ekam ca yo vinŚati ca Śravasyā vaikarṇayojanā vrājānyastaḥ /
Dasmo na sadmanni ŚiŚāti barhiḥ Śūraḥ sargamakṛṇodindra eṣām //
(Ṛgveda – 7.18.11)

Such as 'gaining of ten treasures, ten horses, ten golden pieces (roundish mats), ten chariots with horses and hundred cows' are mentioned in the mantras of *Ṛgveda.* In the *Yajurveda* (1200 - 1000 BCE), the powers of 10 from $10^{º}$ to 10^{12} is listed. [Ek (10º=1), 10^1 (Dasha), 10^2 (Shat) 10^3 (Sahsra - thousand), 10^4 (Ayuta-Ten thousand) ......10^8 (Nyarbud- One hundred millions), 10^9 (Samudra- One thousand millions),10^{10} (Madhya- A ten thousand millions), 10^{11} (Anta- A hundred thousand millions) & 10^{12} (Parardha- a million million or a billion)].

The above number writing system during the *Yajurvedic* period has been mentioned in this table –

Ekam	-	1 (10^0)
DaŚa	-	10 (10^1)
Śatam	-	100 (10^2)
Saḥasram	-	1000 (10^3)
Ayutam	-	10000 (10^4)
Niyutam	-	100000 (10^5)
Prayutam	-	1000000 (10^6)
Arbudam	-	10000000 (10^7)
Nyarbudam	-	100000000 (10^8)
Samudram	-	1000000000 (10^9)
Madhyam	-	10000000000 (10^{10})
Anta	-	100000000000 (10^{11})
Prārddham	-	1000000000000 (10^{12})

In the *Yajurveda*, Even Numbers, Arithmetic Progression (AP) i.e. series of muiltiples of 4 that is also table of 4 [4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 and 48] are mentioned in the *Yajurveda* in this mantra –

CatasraŚca meaṣṭau ca meaṣṭau ca me dvādaŚa ca me ṣoḍaŚa ca me ṣoḍaŚa ca me viṁŚatiŚca me viṁŚatiŚca me caturviṁŚatiŚca me caturviṁŚatiŚca meaṣṭāviṁŚatiŚca meaṣṭāviṁŚatiŚca me dvātriṁŚacca me dvātriṁŚacca me ṣaṭtriṁŚacca me ṣaṭtriṁŚacca me catvāriṁŚacca me catvāriṁŚacca me catuŚcatvāriṁŚacca me catuŚcatvāriṁŚacca meaṣṭācatvāriṁŚacca me yajñena kalpantām//

(Yajurveda – 18.25)

The *Yajurveda* throws light on the series of odd numbers and arithmetic progressions (AP) (i.e. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31 and 33), which are mentioned in this mantra:

Ekā ca me tisraśca me tisraśca me pañca ca me pañca ca me sapta ca me sapta ca me.... ekatriṁśaśca me trayastriṁśaśca me yajñena kalpantām.

(Yajurveda – 18.24)

In the *Yajurveda* (14.23), we also find the four basic operations i.e. addition, subtraction, multiplication, division { +, −, X, ÷ }. In the mantras (5.6.3; 6.4.3; 16.1.2; 9.2.6; 20.7.3 etc) of the *Sāmaveda* **(**1200 - 800 BCE**)**, there is clear mention of numbers.

Taittirīya Saṁhitā **(**1200 to 1000 BCE**),** (7.2.11-20) also mentions the list of numerals –

1, 3, 5, 7, 9, 11, 13, 15, 17, 19..................20, 40, 60, 80, 100 (multiples of 20).

Numbers with the description of 100 and 1000 Ayutam (means 10,000) & Nyarbudam (means ten crores) are mentioned in the following mantra of the *Atharvaveda* **(**1000 - 800 BCE**),** *Atharvaveda* (8.8.7) –

Bṛhatte jālam bṛhat Indra Śūra sahasrārghasya Śatavīryasya/
Tena Śatam sahasramayutam nyarbudam jaghāna Śakro
dasyūnāmabhidhāya senayā //

Vālmīki Rāmāyaṇa (Kāṇḍa VI, Sarga 28, Verse 33-43) mentions not only the strength of Rāma's but also establishes a relation between the numbers -

Śataṁ śatasahasrāṇāṁ koṭimāhurmanīṣiṇaḥ /
śataṁ koṭisahasrāṇāṁ Śaṁkha ityabhidhīyate //

śataṁ śaṁkhasahasrāṇāṁ mahāŚaṁkha iti smṛtaḥ /
maḥāśaṁkhasahasrāṇāṁ śataṁ vṛndamihocyate //

śataṁ vṛndasahasrāṇāṁ mahāvṛndamiti smṛtam /
mahāvṛndasahasrāṇāṁ śataṁ padmamihocyate //

śataṁ padmasahasrāṇāṁ mahāpadmamiti smṛtam /
mahāpadmasahasrāṇāṁ śataṁ kharvamihocyate //

śataṁ kharvasahasrāṇāṁ mahākharvamiti smṛtam /
mahākharvasahasrāṇāṁ samudramabhidhīyate //

śataṁ samudrasāhasraṃ mogha ityabhidhīyate /
śatamoghasahasrāṇāṁ mahaughaḥ iti visṛutaḥ //

evaṁ koṭisahasreṇa śaṁkhāṁ ca śatena ca /
mahāŚaṁkhasaḥasreṇa tathā vṛndaśatena ca //

mahāvṛndasahasreṇa tathā padmaśatena ca /
mahāpadmasahasreṇa tathā kharvaśatena ca //

samudreṇa śatenaiva mahaughena tathaiva ca /
eṣa koṭimaḥaughena samudrasadṛśena ca //

vibhīṣaṇena vīreṇ sacivaih parivāritaḥ /
Sugrīvo vānarendrastvāṁ yuddhārthamabhivartate //

Mahābalavṛto nityam mahābalaparākramaḥ //

An hundred of thousand is called a koṭi by the wise. An hundred of a thousand koṭi is called a Śaṁkha. An hundred of a thousand Śaṁkha is known as a maḥāśaṁkha. An hundred of a thousand Maḥāśaṁkha is here termed a Vṛnda. An hundred of a thousand Vṛnda is known as a Mahāvṛnda. A thousand Mahāvṛnda is called here a

Padma. (An hundredof a thousand Padma is known as a Mahāpadma. An hundred of a thousand Mahāpadma is termed a Kharva. (An hundredof a thousand Kharva is termed a Mahākharva, and hundred of a thousand Mahākharva is termed a Samudra. An hundred of a thousand Samudra is known as a Mahaughaḥ. Thus backed by a thousand koṭi, and an hundred Śaṁkha, and a thousand Mahaughaḥ, and an hundred Vṛnda, and a thousand Mahāvṛinda, and an hundred Padma, and a thousand Mahāpadma and an hundred kharva, and an equal Samudra and an equal Mahaugha, by koṭis of Mahaughas- resembling the sea, and surrounded by the heroic vibhīṣaṇena aas well as his counsellors, that Lord of monkeys, always engirt by a mighty force, and possessed of exceeding strength and prowess, will encounter you in battle.

The above number writing system of the *Vālmīki Rāmāyaṇa* has been mentioned in this table:

1 Koṭi	=	10^7		
10^5 *Koṭi*	=	*Śaṁkha*	=	10^{12}
10^5 *Śaṁkha*	=	*1 Mahāśaṁkha*	=	10^{17}
10^5 *Mahāśaṁkha*	=	*1 Vṛnda*	=	10^{22}
10^5 *Vṛnda*	=	*1 Mahāvṛnda*	=	10^{27}
10^5 *Mahāvṛnda*	=	*1 Padma*	=	10^{32}
10^5 *Padma*	=	*1 Mahāpadma*	=	10^{37}
10^5 *Mahāpadma*	=	*1 Kharva*	=	10^{42}
10^5 *Kharva*	=	*1 Mahākharva*	=	10^{47}
10^3 *Mahākharva*	=	*1 Samudram*	=	10^{50}
10^5 *Samudram (Ogha)*	=	*1 Mahaugha*	=	10^{55}

{1000 Koṭi + 100 Śaṁkha + 1000 Mahāśaṁkha + 100 Vṛnda + 1000 Mahāvṛnda + 100 Padma + 1000 Mahāpadma + 100 Kharva + 100 Samudram + 100 Mahaugha + 1 Koṭi Mahaugha + Vibhīṣaṅa and his four ministers ($10^{10} + 10^{14} + 10^{24} + 10^{30}+10^{34} + 10^{40} + 10^{44} + 10^{52} + 10^{57} + 10^{62} +5$)}

In the *Mahābhārata* (Âdi parva – 2.18-28) mentions numbers of the army of Pānadavas and Kauravas in the battle field of Kurūkṣetra.

The numbers of army was 18 *Akṣauhiṇī.* It was called *Caturaṅgiṇī Senā.*

Chariot – 21,870 X 18	=	3,93,660
Elephant – 21,870 X 18	=	3,93,660
Cavalry – 65,610 X 18	=	11,80,980
Infantry soldiers – 1,09,350 X 18	=	19,68,300

This was the total number of 18 *Akṣauhiṇī Senā.*

In the *Purāṇās (Brahmāṇḍa Purāṇa* - 3.2.92-93; 65.103-104; *Bhāgavad purāṇa* - 9.15.6; 10.61.29 etc) we also find clear reference of the number system.

Four basic mathematical operations (+, -, x, ÷) have been used in the Vedas.

In the *Ṛgveda (7.18.11, 1.45.2, 1.34.11),* we find No.21 as an addition of 20+1, 107 as 100+7 etc. *Ṛgveda (10.72.8, 10.72.9)* mentions 8-7 = 1. *Maitrāyaṇī Saṁhitā* **(**1200 to 1000 BCE**)**, *(1.10.8)* mentions 12x3=36, 12x2=24. *"Dwidhā", "Tridhā" and "Aṁṣa"* etc., words of the Vedas indicate the operation of the division. Fractions are referred to for the first time in the *Ṛgveda (10.90.4)*. As these fractions are called 1/4 (pada), 1/2 (ardha), 3/4 (tri-pada) etc. *Maitrāyaṇī Saṁhitā (3.7.7)* shows the fractions 1/16 (kalā), 1/12 (kuṣṭha), 1/8 (śapha) and 1/4 (pāda). The decimal based system having compound numbers like 11(Ekadasha-Eka+dasha), 21(Ekavimshati-Eka+vimshati), 27 (Saptavimshati-Sapta+vimshati) etc., also indicate the place value system. We find in the symbolic sign of zero (śūnya) in *Ṛgveda* and *"Rupe Śūnyaṁ"* for *prastara* in *Pingalachhandashastra* (200 B.C.). The idea of expressing all quantities by 1-9 figures and every new series of powers of 10 like dasha (2 digit series), shata (3 digit series), sahasra (4 digit series) etc. and *Dvidashati* (*Vimshati*=2x10), *panchdashati* (*panchashat*=5x10) etc. symbolize the concept of zero as integral part of Vedic numeral system. The concept of infinity has been mentioned in the *Yajurveda* and *Brihadaranyakopanishad* (700 BCE), *(3.2.12, 4.1.5)* –

Yatrāyaṁ puruṣo mṛyate kimenaṁ na jahātīti nāmetyanantaṁ vai /
Nāmānantā viśve devā anantameva sa tena lokaṁ jayati //

The Peace invocation of *Ishopanishad* (700 BCE) also highlights the importance of infinity- The invisible is the Infinite, the visible too is the Infinite. From the Infinite, the visible universe of Infinite extension has come out. The Infinite remains the same, even though the Infinite universe has come out of it.

Om pūrṇamadaḥ pūrṇamidaṁ pūrṇāt pūrṇamudacyate /
Pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyate //

This can be presented in mathematical notation as –

$$\infty - \infty = \infty$$

(Infinity) – (Infinity) = (Infinity)

Brahmagupta (598 A.D.), in his book *"Brāhmasphuṭasiddhānta"* has clarified this concept.

Âryabhaṭṭa (476 A.D.) in his book *"Âryabhaṭīyam"* (499 A.D.) has mentioned alphabetical representation of numerals and numbers. He has invented ingenious method to represent alphabetic notation in Gītikāpāda –

vargākṣarāṇi varge avarge avargākṣarāṇi kāt ṅmauyaḥ /
khadvinvake svarā nava varge avarge navāntyavarge vā //
(Gītikāpāda - 2)

The varga letters (from ka to ma are to be written in the place value is (10 raised to the power which is an even number), a square number, the avarga letters (from ya to ha), 10 raised to the power which is an odd number, a non-square number. The varga letters from ka to ma take numerical values from 1 to 25. The numerical value of initial avarga letter ya is 30. Nine vowels are to be written from right to left so that each vowel represents two place values of (powers of 10 raised to even and odd numbers) square and non-square numbers respectively from right to left. This is shown in following table –

Varga **Letters and represented numbers**

Ka-varga	*k-1*	*kha 2*	*g 3*	*gh 4*	*ṅ 5*
Ca varga	*c 6*	*ch 7*	*j 8*	*jh 9*	*ñ 10*
ṭa varga	*ṭa 11*	*ṭh 12*	*ḍ 13*	*ḍh 14*	*ṇ 15*
Ta varga	*t 16*	*th 17*	*d 18*	*dh 19*	*n 20*
Pa varga	*p 21*	*ph 22*	*b 23*	*bh 24*	*m 25*

Avarga

Y	*R*	*L*	*V*	*Ś*	*ṣ*	*S*	*H*
30	*40*	*50*	*60*	*70*	*80*	*90*	*100*

Nine vowels (svara)

A	*I*	*U*	*ṛ*	*ṝ*	*E*	*ai*	*O*	*Au*

In this alphabetic notation, vowels are equal whether short (hrasva) or long (dirgha). As for example, ka = kā = 1, ki = kī =100, ku = kū = 10000 and so on. The numbers can be represented upto 10^{18}.

The velocity of planets in a Yuga *(1 Yuga = 43,20,000 years)* – It is mentioned in the *ślokas* 3 & 4 of *Gītikāpāda*[i] *of Āryabhaṭīya* (the velocity is the the number of revolutions).

For example,

1. *Ravi (the Sun)------- khyaghṛ*

   = kh (u) + y (u) + gh (ṛ)
   = 2 (10^4) + 30 (10^4) + 4 (10^6)
   = 20000 + 300000+ 4000000
   = 43, 20,000

2. *Soma (the Moon)------- cayagiyiṅuśuchṛ□*

   = c (a) + y (a) + g (i) + y (i) + ṅ (u) + ś (u) + ch (ṛ) + l (□)
   = 6 (1) + 30 (1) + 3 (10^2) + 30 (10^2) + 5 (10^4) + 70 (10^4)) + 7 (10^6)) + 50 (10^6)
   = 6 + 30 + 300 + 3000 + 50000 + 700000 + 7000000 +50000000
   = 5, 77, 53, 336

3. *Bhūmi (the Earth)------- niśibuṇ□khṣṛ*

   = ṅ (i) + ś (i) + b (u) + ṇ (□) + (kh +ṣ) ṛ
   = 5 (10^2) + 70 (10^2) + 23 (10^4) + 15 (10^8) + (2+80) X 10^6
   = 500 + 7000 + 230000 + 1500000000 + 82000000
   = 1,58,22,37, 500

There is clear reference of numbers in Āryabhaṭīya. In the Āryabhaṭīya we find eka (1), 10^1 to 10^9 {eka (1), daśa (10), śata (100), sahasra (1000), ayuta (10000), niyuta (100000), prayuta (1000000), koṭi (10000000), arbuda (100000000) and vṛnda (1000000000) } in the following verse –

Ekaṁ daśa ca śataṁ ca sahasramayutaniyute tathā prayutaṁ /
koṭyarbudaṁ ca vṛndaṁ sthānātsthānaṁ daśaguṇaṁ syāt //
(Gītikāpāda - 2)

The value of pi (π) – Āryabhaṭa was the first to mention the most accurate value of Pi (π) which is correct to four decimal places. The ratio of the circumference of a circle to its diameter is a constant, denoted by π. Its value is given by Āryabhaṭa I in the following verse:

Caturadhikaṁ śatamaṣṭaguṇaṁ dwāṣaṣṭistathā sahasrāṇām /
Ayutdwayaviṣkambhasyāsanno vṛttapariṇāḥ //
(Gaṇitapāda - 10)

It means, if we add four (4) to one hundred (100), multiply it by eight (8) and add to sixty two thousand (62000) to that number, the result is approximately the circumference of a circle whose diameter is twenty thousand.

Viz: 100+4=104X8 = 832, Dvāśaṣṭistathā sahasrāṇām = 62,000.
Circumference of the circle = 62000 + 832 = 62832
Diameter = Ayutdvaya = 10000X2 = 20,000

$$\text{The value of } \pi = \frac{\text{Circumference}}{\text{Diameter}} = \frac{62{,}832}{20{,}000} = 3.1416$$

This value of π is accurate to within 0.00024%.

Indeterminate Equations of the First Degree : Kuṭṭaka

We find the trace of indeterminate equations from the time of *śulvasūtras* (800-400 BCE). Greek mathematician Diophantus (3rd century) is given credit for solving indeterminate equations. The problem of finding solution in integers for *X and Y* in an equation of the from :- $ax + c = by$ where a, b and c are integers and it was given great importance by ancient mathematicians and astronomers. *Āryabhaṭa* was the first mathematician who solved indeterminate equations in integers in a systematic method. He also used it to solve the problems of determining the periods of the Sun, the Moon and thse planets in astronomy[ii]. The method of general solution of indeterminate equations of first degree in positive integers developed by *Āryabhaṭa* is called *Kuṭṭaka* which literally means breaking or pulverizing. *Bhāskara* I who has explained the method elaborately with examples in his commentary on the *Āryabhaṭīya.*

Example :- Find the number which gives 5 as the remainder divided by 8, 4 as the remainder when divided by 9 and 1 as the remainder when divided by 7.

The problem is expressed algebraically in the following equation:- $N = 8x+5 = 9y+4 = 7z + 1$

By the method of *Kuṭṭaka,* we get the least value of unknown number **N** is 85.

Quadratic Equations

Āryabhaṭa formulated the method for calculation of compound interest which provided the solution of quadratic equations firstly. Later, *Shridharacharya* (750 AD) elaborated the method for solving quadratic equation ($ax^2 + 2bx = c$). *Āryabhaṭa* says that the problem is "a principal amount (A) is lent for unknown monthly interest (x) and the unknown interest is lent out for interest for some period equal to (B). What is the rate of interest (x) on the principal amount (A)." *Āryabhaṭa* gave the formula "multiply the sum of the interest on the principal and the interest on this interest by the time and by the principal. Add to this result the square of half the principal. Take the square root of this. Subtract half the principal and divided the remainder by the time. The result will be the interest on the principal." This formula involves the solution of a quadratic equation in the form of $ax^2 + 2bx = c$.

The solution in modern notation is, $x = \dfrac{\sqrt{B \times A \times T + \left(\frac{A}{2}\right)^2} - \left(\frac{A}{2}\right)}{T}$

For example, the sum of 100 (A) is lent for one month. Then the interest received is lent for six months (T). At that time, the original interest plus the interest on this interest amounts to 16 (B).

$$x = \frac{\sqrt{16 \times 100 \times 6 + \left(\frac{100}{2}\right)^2} - \left(\frac{100}{2}\right)}{6} = 10$$

The interest received on principal 100 in one month is10.

Positions and place values of digits in *Līlāvatī (Verse - 12)* of *Bhāskarācārya* (1114 A.D.) have also been mentioned.

- Indispensability word signifies the importance of number & numerals in the realm of Mathematics. Without knowing number & numerals we can't calculate and integrate anything.
- The origin of place value may be traced out from the Ṛgveda, Maitrāyaṇī Saṁhitā and Taittirīya Saṁhitā.

Conclusion

So, the numbers and numerals of Indian Intellectual Traditions deal with relevant issues like – how can we make available our traditional knowledge of Mathematics for the students as well as teachers of Sanskrit and modern Science and to integrate this untapped valuable treasure with the curriculum and syllabus for different institutions. Teaching learning community may derive information on Mathematics and other disciplines and integrate those with modern day information. It may be useful in interdisciplinary courses for the students of Schools, Colleges and Universities in India and abroad.

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