# Relevance of Shulbasūtras of the Yajurveda: Modern Context

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**ABSTRACT**

The Yajurveda (1200 BCE to 1000 BCE) contains valuable knowledge in the realm of mathematics which has eternal value. The science of mathematics with all its branches such as arithmetic, algebra, geometry and trigonometry, etc., was so well developed in ancient India that many modern scholars find to their dismay that some of the European discoveries were discovered in previous centuries. It is necessary to integrate this valuable treasure of Vedic mathematical knowledge with that of modern mathematics. Vedic people were fully acquainted with mathematical knowledge and had knowledge of applied geometry as well. The origin of geometry is from Shulbashutra. Vedic people used to make sacrificial altars in definite prescribed shapes and sizes using special types of bricks. There are eight Shulbashutras- Baudhayana, Manava, Apastamba, Katyayana, Satyashadha, Vadhula, Varaha and Maitrayani. The first four of these are available as independent texts. Seven Shulbashutras belong to the Krishna Yajurveda and one, namely Katyayana Shulbashutra, belongs to the Shukla Yajurveda. Shulbashutras of vedanga taught the important rules to construct triangles, rectangles, squares, parallelograms and circles by explaining their properties.

The equation $c^2=a^2+b^2$, which is known as Pythogoras theorem, was first given by Baudhayana (800 BCE) before Pyhthagoras (580-500 BCE) . This is basically Shulba theorem (Baudhayana Shulbashutra: 1-48) .

This research paper is mainly focused at Shulbashutras in determining the special blend of scientific approach of Vedic seers in the field of modern mathematics.

The importance of mathematics has been highlighted in the Vedanga Jyotisha (1400 B.C.) of Lagadha –

*Yathā śhikhā mayūrāṇāṁ nāgānāṁ maṇayo yathā,*

*Tadvat vedāṅga śāstrāṇāṁ gaṇitaṁ mūrdhani sthitam.*

Vedangajyoisha ; 4

i.e. 'Like the crests on the heads of peacocks, like the gems on the heads of the cobras, Mathematics is at the top of the Vedanga Shastras'.

Six supplementary sciences of Veda are known as the Vedangas and the oldest enumeration occurs in the Mundakopanishad (I.1.5). They are: Śikṣā (phonetics or pronunciation, Kalpa (ritual), Vyākaraṇa (grammar), Nirukta (etymology), Chandas (metre) and Jyotiṣa (astrology or astronomy). They were the subjects of instructions to understand the Vedic texts. Their origin seems to be in the Brāhmṇas and Âraṇyakas where, along with the explanation of the sacrificial ritual, we find occasional discussions on all these subjects. Later on, arose separate special schools within the Vedic schools which systematised them in the form of texts in the unique aphoristic prose called the sūtras. The Kalpasūtras (the oldest), their chief contents were the rituals (kalpa) directly connected with the Brāhmṇas and Âraṇyakas. They contains the rules for the sacrificial rituals for all practical purposes. They are divided into four parts, Śrautasūtras - sacrifices for the Brāhmṇas, Gṛhya-sūtras - dealing with domestic ceremonials and sacrifices, Dharma-sūtras - directly connected with Gṛhya-sūtras and Shulbasūtras - directly connected with Śrautasūtras.

Śrautasūtras prescribe rules for the laying of three sacrificial fires - Âhvaniya (altar in the form of square), Garhpatya (altar in the form of circle) and Dākṣiṇātya (altar in the form of semi-circular) for Agnihotra, Daś-pūrṇamasa sacrifices, etc. Śulva means to measure and the noun Shulba means rope or chord which was used on construction of yajñas (sacrifices) and sūtra means rules. Even today in India, masons use a thread or chord in the construction of houses. The Shulbasūtras contain precise rules for the measurement of the building of the place of sacrifice (yajña) and the fire altars. The vedis or fire altars are known as "chitis."

We find the measure of Mahāvedi or the Great Altar in the following figure given by Swami Satya Prakash Sarasvati in his book *Founders of Sciences in Ancient India (Part- II)* on Page- 618:

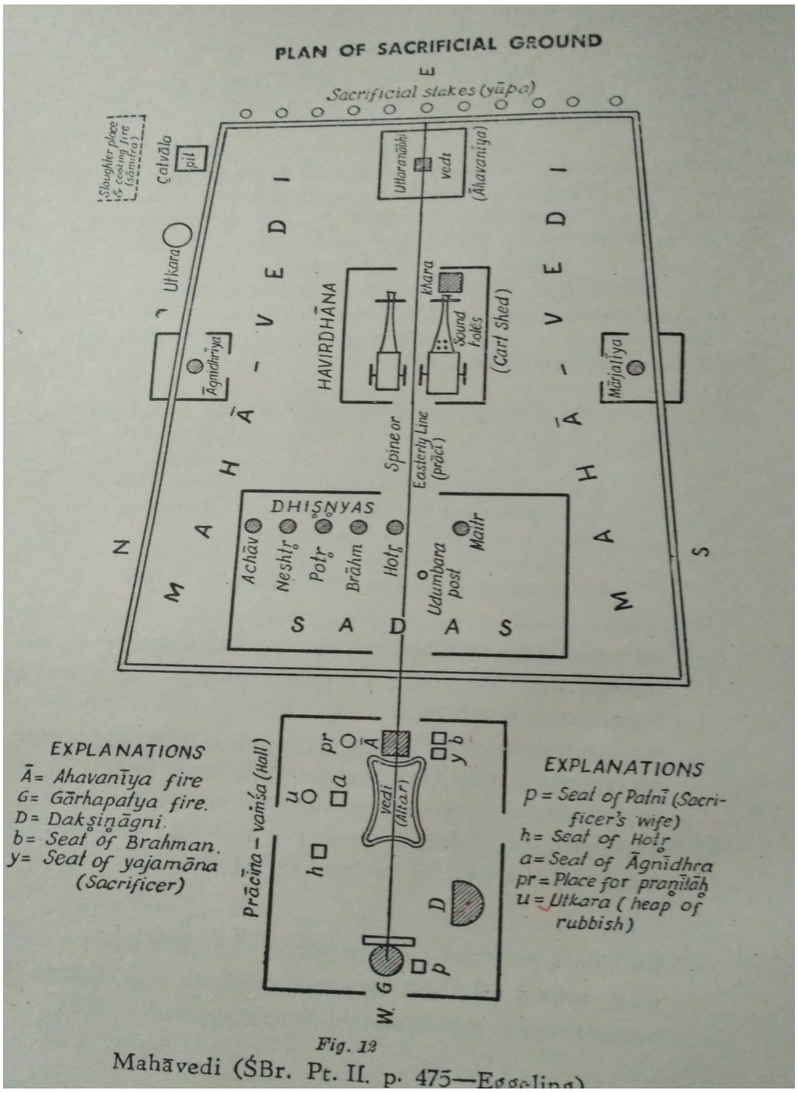
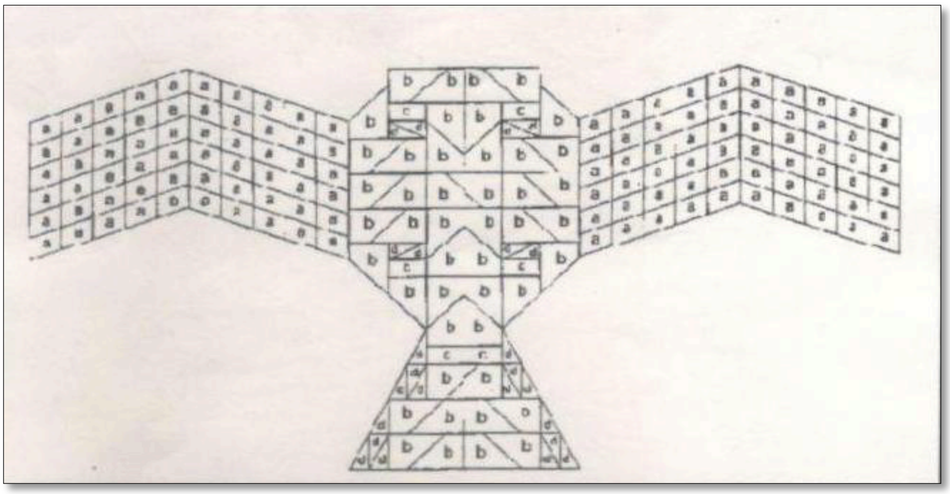


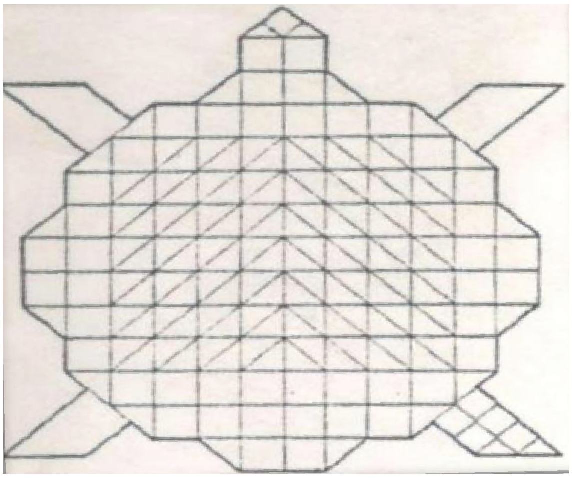
*Fig. 12*
Mahāvedi (ŚBr. Pt. II. p. 475—Eggeling)

Geometries of Shulbasūtras were used in the religious works. It is said that for the desire of heaven construct fire altars in the form of falcon (shyena).



The first layer of the vedi in the form of a falcon (Garuda) is shown above. Every wing has a type of 60 bricks and in the middle 'b' type of 46 'c' type of 'd' type of 24 bricks.

To win the world of Brahman one should construct a fire-altar in the form of a tortoise.



To destroy existing and future enemies one should construct a fire-altar in the form of a rhombus.

**FRACTIONS IN THE SHULBASŪTRAS (GEOMETRIC SOLUTIONS FOR ALGEBRAIC PROBLEMS)**

Shulbasūtras have many examples for the operation of fractions to calculate the sizes of different shaped bricks and the area of different parts of Shyena, Rathachakra and Kurma altars etc. For the solution of linear equations, Baudhayan has given many methods as quadratic equations of the form

$$ax^2 = c \text{ and } ax^2 + bx = c$$

Solutions of indeterminate equations of the first degree has been described by Baudhayana. Later in Indian mathematics such indeterminate equations are called Kuttaka. First degree indeterminate equations were systematically discussed firstly by Aryabhatt I (476 A.D.). Baudhayana prescribes that a *Gārhpatya* vedi should be constructed with five layers of bricks, each layer containing 21 bricks so arranged that their rift into two consecutive layers do not concide. These qualities of

*Gārhpatya* lead to simultaneous indeterminate equations of the first degree. If each layer has *x* square bricks of length *1/m* unit each and *y* square bricks of length 1/n unit each with *m* >*n*,

$$x + y = 21$$

$$\frac{x}{m^2} + \frac{y}{n^2} = 1$$

There may be two sets of positive integral solutions as $x = 16,\ y = 5,\ m = 6,\ n = 3$ and $x = 9,\ y = 12,\ m = 6,\ n = 4$ .

Baudhayana obtained these positive integral solutions. He says that firstly make 3 types of bricks having lengths of 1/6,1/4 and 1/3 units. In first, third and fifth layers, place 9 bricks having length of 1/6 and 12 bricks having length of ¼ unit. In second and fourth layers place 16 bricks having length of 1/6 unit and 5 bricks having length of 1/3 unit.

In the specification of Shyena chiti (falcon shaped altar), Baudhayana says that it has the area of 7½ purushas (900 angulas) and each layer consists of 200 bricks which leads to the simultaneous indeterminate equation, i.e.,

$$x + y + z + u = 200, \quad \frac{x}{m} + \frac{y}{n} + \frac{z}{p} + \frac{u}{q} = 7\tfrac{1}{2}$$

## FIRST CONCEPT OF CYCLIC QUADRILATERAL

Baudhayana, in the construction of Rathachakra chiti (chariot wheel altar) propounded the technique of inscribing a square as large as possible within a circle of given diameter.

Elementary trigonometry also existed in the Shulbasūtras as in a procedure for circling the square and sine of $\pi/4$ (= $45^0$) as $1/\sqrt{2}$.

## THE VALUE OF SQUARE-ROOT OF 2

The value of square-root of 2 and square root of 3 has been mentioned in the Śulbasūtras which is accurate up to 5 decimal places.

Baudhayana used irrational numbers. By using the fractions and surds for irrational √2, he provided a good rational approximation in the following form:

> *"Samasya dwikaraṇi pramāṇam tṛtīyena vardhayettacca*
>
> *caturthenātmacatustriṃśonena saviśeṣaḥ".*

(Baudhayana Shulbasūtra;I-61-62;Apastamba Shulbasūtra; I-11-12)

i.e., Increase the measure by its third and this third by its own fourth less the thirty fourth part of the fourth. This is the value with a special quantity in excess. If we take 1 for the measure of a square side, the above formula provides the diagonal as follows:

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3x4} + \frac{1}{3x4x34} = \frac{577}{408}$$

$\frac{577}{408}$ = 1.41421568627451 (which is correct to 5 decimal places)

In the same manner, the value of tri-karṇi,

√3=1+2/3+1/3x5-1/3x5x52=1.7320513

We see that G.Thibaut, Burk Muller, B.B.Datta have reconstructed possible methods for the above calculations.

**VALUE OF PI (Π)**

Baudhayana gave approximations for value of $\pi$ for constructing circular shapes. His approximation for $\pi$ is equal to 3.088.

For squaring of a circle (where *r* = radius, *d* = diameter)

1. Method for desired Square's side= 7/8d + d/8- (28d/8x29 + d/8x29x6-d/8x29x6x8)
2. Methods for desired Square's side= d-2/15d = 26r/15

As Baudhayana mentions,

*Yupāvaṭāḥ padaviṣkambhāḥ, tripadapariṇāhāni yūpoparāṇi*

(Baudhayana Shulbasūtra,4.112-113)

Mānava Shulbasūtra states better approximation of $\pi$ as 3.125 or 3.1604.

Archimedes (287-212 BCE) accepted the value of $\pi$ as 22/7 and 223/71 i.e. between 3.1428 and 3.1408.

According to Âryabhatt (476 AD), $\pi$ = 3.1416 (Aryabhaṭīya, Ganitapada, sloka 10), which is universally accepted today.

**THEOREM OF PYTHAGORAS PREDATED BY BAUDHĀYANA**

Vedic people were fully acquainted and had knowledge of applied geometry as well. The origin of geometry may have been the Shulbasūtra.

The equation $AC^2 = AB^2 + BC^2$, which is known as Pythagoras Theorem, was firstly given by Baudhāyana (800 BC) before Pythāgoras (580-500 BC). Baudhāyana was not first to propound methods of theorems and system of numbers. He also mentions his indebtedness to the earlier works such as Taittiriyasamhita and Shatapath-Brahman. In fact the Pythagoren triples, $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$, $8^2 + 15^2 = 17^2$, $7^2 + 24^2 = 25^2$, $12^2 + 35^2 = 37^2$ and $15^2 + 36^2 = 39^2$, are mentioned in Taittiriyasamhita (5.2.3) and in Shatapath-Brahman.

This is basically Shulba Theorem –

*Dirghacaturasrasyakśṇayā rajjuḥ pārśvamānī, tiryaṅgmānī ca yatpṛathagbhūte kurutastadubhayaṁ karoti.*

(Baudhayana Shulbasūtra; I.48)

i.e. the (square) areas produced separately by the length and the breadth of a rectangle together equal the area (of the square) produced by the diagonals.

Thus we see that the theorem of Pythagoras was already mentioned in Baudhāyana Śulbasūtra. It is also mentioned in the Âpastamba Śulbasūtra (1.4), Kātyāyana Śulbasūtra (2.7) and Mānava Śulbasūtra (10.10). However, the earliest evidence of Pythagorean triples appears on the famous Plimpton 322 tablet from Babylonia (present-day Iraq) dating back to 1800 BC.

Tributes to Ancient Indian Mathematics

The French mathematician Pierro Simon Laplace (1749-1827) said "*it is India that gave us the ingenious method of expressing all numbers by means of 10 symbols, each symbol receiving a value of position as well as an absolute value. The idea escaped the genius of Archimedes and Apollonius*". Albert Einstein has marked the Indian contribution *"We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made."* Indian Scientist Dr A.P.J.Abdul Kalam has written about the importance of ancient Sanskrit literature - "*Ancient Sanskrit literature is a store-house of scientific principles and methodology. The work of our ancient scholars should be thoroughly examined and where possible integrated with modern science".* (Ignited Minds, P.87)

## CONCLUSION

Thus, Shulbasūtras of the Yajurveda are relevant in understanding the origins of mathematics. Due to the practical needs for the results of 'theorems' it is likely that the requirement for proofs was omitted. In a modern-day context there are ongoing questions as to how these ancient mathematicians arrived at their conclusions. It is clear that the Shulbasūtras provide a glimpse into the considerable achievements of mathematicians in ancient India. Yet there is still a vast storehouse untapped. From the 'schools' of mathematics in Kerala, particularly from $13^{th}$ C to $16^{th}$ C AD, there is a treasure trove of advanced material, as yet untranslated, and there is a definite need for research into this. Just as knowledge of ancient herbal medicines can be forgotten and, if rediscovered, can be of great value, so too can ancient mathematical knowledge be refound and put to use in modern day mathematics.

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