Contributions of Kerala to Mathematics

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ABSTRACT

The discovery of general algorithms of calculus and the development and application of infinite series techniques are major tools for modern mathematics. The astronomers and mathematicians from Kerala made major contributions during 14^{th} to 16^{th} centuries. And these contributions are 300 years prior to European mathematicians' statements in these fields.

Social background and political situations helped in these contributions. Mathematics was an important and helpful tool to the study of Jyotisha sastra. The result was that those mathematical topics like properties of a circle, sphere, mensuruation, etc., needed in the study of astronomy got prime attention and gained enormous development. The Bhutha Samkhya and Katapayadi were the numeration systems used in ancient texts. The major mathematical texts from Kerala can be classified into two periods, 7th and 14th centuries. Their works were mainly commentaries, improvements on the works of earlier scholars. The period 1350-1600 was the glorious period when Kerala Mathematics attained great heights known as Golden age of Kerala Mathematics. The study of the circle and its chords broke the finite barrier and searched for the infinite. The significant period in the history of Kerala Mathematics started with Sangama Gramma Madhavan (1340-1425). Madhavan gave the value of Pi(π) correct to eleven decimal places. He described it in the Bhuta Samkhya system. Sangama Gramma Madhavan also stated

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$$

He also made sine tables from zero degrees to ninety at the intervals of 3.75 degrees.

Madhavan's main contributions came to be known through the works of his followers, such as Neelakanda Somayaji, Jeyshtadevan, Sankaravaryar etc. The knowledge might have been transmitted to other parts of the world through foreigners that came to Kerala for trading of spices, etc. Many of the results discovered by Kerala Mathematicians became known as the works of a few Europeans.

Key words: Kerala Mathematics, *Katapayadi, Bhuta Samkhya, Sangama Gramma Madhavan*

INTRODUCTION

Two powerful tools contributed to the creation of modern Mathematics in the seventeenth century; the discovery of general algorithms of calculus and the development and application of infinite series techniques. In these topics, Kerala, especially 500 square kilometers north of Kochi made remarkable contributions to these topics during 14^{th} and 16^{th} Centuries. Importantly, these discoveries were at least 300 years prior to similar works of European Mathematicians like James Gregory, Issac Newton, Leibniz, etc.

SOCIAL BACKGROUND AFFECTING DEVELOPMENTS IN KERALA

A retrospection of the social and cultural background of Kerala in the early medieval period, that is, from the 9^{th} to 16^{th} century, may be helpful in understanding the extraordinary growth of Mathematics and astronomy in Kerala.

There was a slow but steady southward movement of Brahmin and Brahmin culture, migrating from river valley to river valley since the days of the *Mauryan* Empire. The consequent synthesis between Aryan Vedic Brahmin culture and the non-Brahmin and non-Aryan cultures gave birth to the celebrated ancient Indian Civilization. In the course of this long drawn out process we can find the emergence of regional sub-national cultures. An important phenomenon at the core of the evolution is the large number of land granted to Brahmins by kings in each region. This resulted in temple centered Brahmin settlements known as *Agrahara*s. Such settlements can be seen in Kerala, the south-west end of peninsular migration route also. Apart from these, in Kerala, there were Brahmin settlements where they themselves took the initiative to clear the thick forests with the help of local tribes and organized the settlements, with the *Sabha*s administering self-governing villages. They failed to maintain self-government and finally invited the Prince from the land beyond the Western Ghats to rule the country with their advice and guidance. Thus a new Cera dynasty with Perumal in the capital at Mahodayapuram or Thiruvanchikulam (present Kodungallur north of Kochi) came into existence. Even after the decline of the Ceras their successors continued to hold sway in different parts of Kerala. The last of the Perumals was Bhaskara Ravi Varma. Ancestors of Cochin Kings, as descendents of Perumals, continued to reside at Mahodayapuram. Through the gradual merger of smaller territories Cochin gained size and status. Until the dawn of 15th century, Kodungallur was the capital of Cochin State and later Cochin City became capital and Thripunithura the royal township. Even though the ritual monarchy of the Ceraman Perumals came to an end at the beginning of 12^{th} Century the strong Brahmin command continued to flourish until the 19^{th} Century when British rule was established. There was uninterrupted continuity of culture and education which favoured growth of Scientific research.

An important factor that helped the growth of scientific tradition was the freedom from military invasions from the North which had uprooted all native dynasties and destroyed traditional religion and culture. Fortunately, Kerala was spared, thanks to the Western Ghats.

Another positive factor was the flourishing sea trade of Kerala with its numerous natural harbours. From the beginning of the Christian era the Greek and Roman sailors frequented Kerala ports. Kerala had a world monopoly of pepper which European countries required in large quantities to tide over the winter challenge. Sandalwood, teak, and all sorts of spices, perfumes and medicinal plants were available in Kerala. These attracted traders from East and West - Jews, Syrians, Chinese, Arabs, etc. The rulers and people of Kerala welcomed all of them in spite of differences in faith, language, customs and costumes. Continuous exposure to the cultural developments in the East and West must have widened the mental horizon and increased the intellectual capacity of the enlightened sections of people.

ASTRONOMICAL FOUNDATIONS

From early times people of Kerala held the study of Celestial luminaries - Jyoitisha sastra, in high esteem. Jyotisha sastra comprises of two parts, a theoretical part and practical part. The phase of moon, solar and lunar eclipse and variations in the movements of planets, etc., are some of the events which could be calculated in advance. These, as well as descriptive details pertaining to earth and other celestial bodies, belong to the theoretical part. Forecasting future events, reckoning of auspicious moments –Muhurthams, etc., fall within the scope of prediction part. Some people use Astonomy and Astrology to denote the theoretical part and prediction part respectively.

There is evidence to show that the Aryabhatiya tradition of Astronomy was very strong in Kerala, at least by the 9th century. Sankaranarayanan, the court astronomer of Ravi Kulasekhara, composed Vivarana, a commentary of Laghubhaskariya. Though the original text uses the Bhutasankhya system to suggest dates, the Kerala commentators used the Kadapayadi system of letter numerals which was popular in Kerala. From the 9th century inscriptions in temples in Kerala refer to solar months, dates, weeks and Naksatras, besides the other details. This is also found in other parts of South India, but the speciality of Kerala is that several texts contain the position of Jupiter (Brahaspati or Guru- Vyazham in Malayalam). Special importance was attached to this planet in Kerala. The Grand festival Mamankam, was celebrated once every twelve years. This is based on the Jupiter cycle. Perumals were initially allotted a twelve-year term of rule. Land records were renewed after twelve years in traditional society. The Saka era, Kali era and Kollam era were used in Kerala.

Mathematics was an important and helpful tool to the study of Jyotisha sastra and fostered as such. The result was that those mathematical topics, like properties of circle, sphere, mensuration, etc., needed in the study of astronomy got prime attention and gained enormous development.

NUMERATION SYSTEMS

Indians had separate names for the powers of ten,

Eka = 1, Dasa= 10, Sata = 100, Sahasra = 1000, Ayuta = 10 000, Niyuta = Laksha= 100000 Prayuta = 1000000.

Even though several systems of numeration sprouted in ancient India, only few gained popularity and acceptability. The Bhutha Samkhya system and Kadapayadi system were two systems which survived for long time.

Bhutha Samkhya

Bhutha Samkhya system is the older system. As of now, we could not identify the individual or the first text in which the system used first time. Researches show that the Bhutha Samkhya system was in use as early as first century of Common era (AD). Samkhya means a number and Bhuta-element, part, component, etc. In this system numbers are indicated by well-known objects or concepts having as many parts or components as the numbers they represent

For example:

0 is denoted by sunya (Void) kha = sky antariksha = atmosphere, purna = whole, randra = hole,

1 is denoted by sasi = moon, bhumi = earth, etc.,

2 is denoted by netra = eyes, bahu = hands, karna = ears, paksha = moon's waxing and waning periods, etc.,

3 is denoted by kala = time, past, present and future, loka = heaven, earth and hell,

4 is denoted by Veda - Rig, Yajur, Sama and Atharva, dik or dis = directions; east west, north and south,

5 is denoted by bhuta – Five elements (Ehter, Air, Fire Water and Earth), Pandavas; each of which have five members,

12 is denoted by rasi = signs of zodiac,

32 is denoted by dants = teeth, and so on.

Any synonym of a word denotes the same number.

The principle of place value was used but the mode of writing was right to left.

Netra –kala – yuga = 432

Masa kala veda danta = 32 4 3 12

pandava dantam = 325

Sunya kha purna randhra netra = 2 0000

Mathematicians and Astronomers of India composed their works in verse too. The multiple words corresponding to each number gave them sufficient choice to select. This made the versification simple and easy to compose as well as to remember. To decode the expressions to their numerical forms one would have to be familiar with concepts such as yugas and ancient holy books of India. The same word sometimes stood for two and occasionally for more than two different numbers eg: paksha for 2 as well as 15, loka for 3 as well as 14, dik for 4 as well as 8 and 10.

The ambiguities must have created some confusion and also went against the universal acceptance. The Bhutasamkhya system was also apparant in some of Kerala works belonging to later periods.

Katapayadi System

The starting point of this system is the Sanskrit alphabet. Sanskrit was the language of the scholarly section throughout India and Malayalam is the regional language of Kerala. All letters of Sanskrit alphabet also occur as such in Malayalam. The subdivisions into consonants and vowels, etc., and pronounciations are also identical. The only difference is in the scripts.

In this system the digits 1 to 9 and 0 are denoted by the consonants as indicated below:

Katapayadi

1	2	3	4	5	6	7	8	9	0
ka	kha	ga	gha	nga	ca	cha	ja	jha	nja
Ta	Tha	Da	Dha	Na	ta	tha	da	dha	na
pa	pha	ba	bha	ma					
ya	ra	la	va	sha	Sha	sa	ha		

Vowels following consonants have no special value. So each of letters k, ka, ki, ku denotes the same number – 1.

The name Katapayadi derives its name from k, t, p and y the first letters in the first column of the table given above.

The system follows decimal notation, a right to left arrangement. The first letter in the word stands for unit place digit. The second letter stands for ten’s place digit and third for hundredth’ place and so on.

Eg; Ma dha va (Ma = 5, dha = 9 ,va = 4) indicates the number 495.

Nananana raga (na = 0, ra =2 ga =3) indicates 32 0000

The special advantage of this system is that numbers running to several digits can be rendered into meaningful expressions, which are easy to remember. The mnemonic of any number is called *paralperu*.

The flexibility in the choice of letters, consonants and vowels following them for each digit, makes it possible to frame several different expressions for the same number. This flexibility is much better than Bhutasamkhya.

The general belief is that the astronomer *Vararuchi* was the proponent of the Katapayadi system. From the circumstantial evidence, historians believe that Vararuchi lived in the 4th century AD. In due course of time the system became popular throughout South India. This system is believed to be one of the major contributions of Kerala to Mathematics.

Vararuchi has another feather in his cap. He composed *Candra vakyam* –the 248 vakyas give the positions of the rising moon on as many consecutive days. They were explained in kadapayadi format. These Candra vakyas were used by scholars from 4th century onwards.

MATHEMATICIANS OF KERALA

7th to 14th Century Mathematicians of Kerala

Haridatta (650-700 AD), *Govindaswami* (800-850), *Sankaranarayanan* (825-900) and Talakkulam *Govinda Bhattathiri* (1237-1295) are the known Mathematicians or astronomers of these seven centuries.

Their works were mainly commentaries, improvements on the works of earlier scholars. Haridatta modified the Aryabhateeya methods to find the planetary positions. He used Katapayadi system to simplify the methods. This simple method was known as Parahita system. This was followed to perform religious rituals. These include birthdays, death anniversary, upanayanam (initiation of a boy to Brahmin way of life) and *muhurtham* for marriages, etc.

These refinements were well accepted due to its simplicity and accuracy. The *Parahita* system became popular in Kerala and gradually spread to other parts of south Indian also.

Sankaranarayanan enjoyed the patronage of Cera King *Ravi Varma Kulasekharan* of Mahodayapuram. Sankaranarayanan was the in charge of Observatory at Mahodayapuram.

Govinda Bhattathiri was a bright star in the galaxy of Kerala astrologers.

Malayalam Calendar

The year 825 AD was a historic one for Kerala astronomers. In August of that year a new era was launched. It was called Kolla Varsham (Malabar era-ME, in English). Ravi Varma Kulasekhara was the Cera king at that time. He himself was an astronomer and mathematician and wrote treatises. Ancient astronomers of many civilizations observed that the moon and planets were never at very great angular distances from the ecliptic (apparent annual path of sun). They imagined a belt in the sky extending to eight degrees on either side of ecliptic and called it zodiac. Twelve groups of stars or constellations were spotted in the zodiac. Further, they noticed the 30-day period the sun takes to travel through each constellation. The constellations were named according to their shapes. These are known as sign of Zodiac. The Kerala term for sign is Rasi. Kerala astronomers considered each Rasi as an arc of a circle, facing an angle of 30 degrees at the centre.

The Golden Age

The period 1350-1600 was the glorious period when Kerala Mathematics attained great heights. The study of the circle and its chords broke the finite barrier. Most of the innovators did not bother to spread these findings. These manuscripts written in palm leaves were lying in palaces or illams as private collections. These were accessible only by a select few people. That is the one reason many historians of mathematics assume that nothing much happened after the 12th century in Indian mathematics.

KERALA SCHOOL OF MATHEMATICS

Madhavan of Sangama Gramma

The significant period in the history of Kerala mathematics started with Sangama Gramma Madhavan (1340-1425). The birth place of Madhava, Sangamagrama*,* is now known as Kallettumkara, near Irinjalakuda. Venuaroha is an important work of Madhava. In this he mentions belonging to *Bahuladhisthita Bihara*, which is the Sanskrit form of iranni (elangi tree) ninna palli in Malayalam.

Later writers also refer to Madhavan as golavid (expert in spherics). The time of his writings can be verified in his works like Sphuta Candrapti. The date corresponds to 1400AD. The works of scholars who followed him also confirm the period. Madhavan's discoveries are known to us mainly through the statements and works of his followers.

Madhavan gave the value of Pi(π) correct to eleven decimal places. He described it using the Bhuta Samkhya system.

This given as the circumference of a circle having diameter 9×10^{11} units is 2827433388233.

As per this calculation the approximate value of pie is 3.1415 9265359.

Sangama Gramma Madhavan also stated $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$

This series was known as the Leibniz (1646-1716) series.

The findings of sine and cosine series in trigonometry are another contribution by Madhavan which made him one of the most eminent mathematicians of the world. He made sine tables from zero to ninety degrees at the interval of 3.75 degrees. Madhavan's main contributions came to be known through the works of his followers like Neelakanda Somayaji, Jeyshtadevan, Sankaravaryar, etc.

Paramesvaran Namputiri

Paramesvaran Namputiri (1360-1460) belonged to Vataserri illam of Alathiyur in Malappuram. The illam stood on the northern bank of the river Nila (Bharatha Puzha) close to where it merges with the Arabian Sea. Drg Ganitha and Gola Dipika are the major works of Parameshwaran. He evolved the Drk system for astronomical calculations with more accuracy. This was the result of his astronomical studies coupled with observations of celestial bodies for more than fifty years. This system was useful to predict eclipses, etc. Parameshwaran observed and recorded several solar and lunar eclipses from 1393 AD onwards.

Parameshwaran also put forward the formula for the radius of the circle circumscribing a cyclic quadrilateral in terms of its sides, in his commentary on Lilavati.

If a, b, c, d are the sides of the quadrilateral, the semiperimeter – $S = (a + b + c + d)/2$ and

R the radius of the circle circumscribing it = $\frac{1}{4} \sqrt{(ab+cd)(ac+bd)(ad+bc)/(s-a)(s-b)(s-c)(s-d)}$.

The credit of this discovery is normally given to Simon Antoninie Jean Lhuilier, who published it in 1782 AD.

Vatasseri Damodaran

Damodaran (1410-1510) was the son and student of Parameshwaran. He himself was a famous teacher. His disciple, Nilakanda Somayaji, quotes several excerpts from Damodaran's works. Damodaran can be considered as the link connecting Parameshwaran to Nilakanda in the teacher-student chain carrying the torch of celestial studies.

Nilakanda Somayaji

Nilakanda Somayaji (1443-1545) belonged to Kelallur illam of Trikkandiyur near Tirur in Malappuram. Trikkandiyur was a famous centre of learning.

Tantra Sangraha is his major work; believed to be written in 1500 AD. It is a fully-fledged text on Astronomy. The work contains 431 verses divided into eight chapters. There are several mathematical results. Tantra Sangraha deals with Spherical triangles formed by the Sun, North Pole and zenith on the celestial sphere, which are called astronomical triangles. Tantra Sangraha also gives several methods related to calculus.

Some of the interesting topics include the method to find sum of an arithmetic progression and the representation of series using rectangular strips.

Nilakanda's Geometrical Demonstration of Sums of Series of Natural Numbers

Sum of Arithmetic series

The terms of an arithmetic series are represented by rectangular strips. The lengths of strips are set equal to the magnitude of the terms and breadth of each strip is unity.

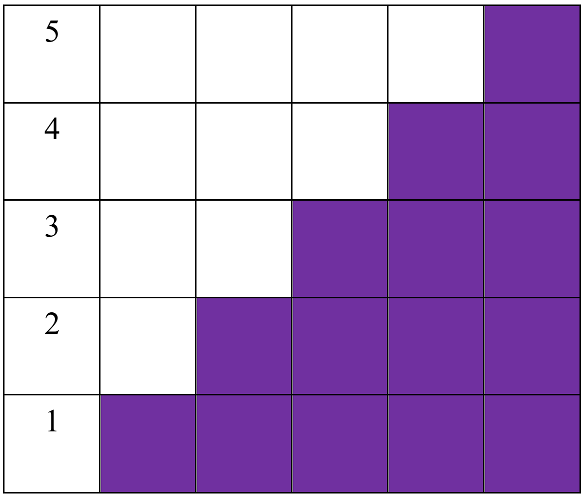
The rectangular strips represent the successive terms of the series. They are piled up side by side in the order. The figure so formed is called *Sredhikshetra* or a *series figure*. The area gives the sum (*Sn*) of the series. Two identical sredhikshetras representing *Sn* are joined together by arranging the second one in the reverse order. Now these Sredhikshetras together form a larger rectangle. The length of the rectangle is equal to the sum of the first and the last terms (*a* + l), and breadth equal to the number of terms in the series (*n*).

The area of the rectangle is $(a+l)\times n = 2\times S_n$ (Area of two Sredhikshetras).

$$\text{So } S_n = \frac{n}{2}\times(a+l)$$

Geometrical demonstration of Sums of series of Natural Number

Eg: To find sum of 1+ 2 + 3 + 4+ 5



In the figure Sn = 5/2 (1 + 5) = 15

Nilakanda travelled almost the entire state to collect manuscripts. He was also in contact with scholars outside Kerala. The work titled Sundararaja Prasnotharam is a collection of Neelakanda's clarifications on the doubt raised by Sundararajan, an astronomer of Tamil Nadu.

Nilakanda Somayajee's other works include Gola Sara - Spherical astronomy, a primer, Siddhanta-Darpana - the mirror of the laws of astronomy, Chandrachaya Ganitha - computation of time from the shadow cast by the moon, Grahananirnaya - computation of lunar and solar eclipses, Graha Pariksakrama - principles and methods of astronomical computation from direct observation.

Nilakanda Somayajee was reputed to be an excellent teacher. Jyeshta Devan and Sankara Varior were his disciples. Chithrabanu who wrote Karanamrutham in 1530, was also a disciple of Nilakanda. The Karanamrutham contains solutions for simultaneous equations in two unknowns.

Narayanan a disciple of Chithrabhanu wrote *Kriyakramakari* in 1556. It is a commentary on Lilavati of Bhascaracharya. In fact Sankara Varior of Thrikkattiri in Ottapalam started the work and Narayana completed it.

Jyestha Devan

Jyestha Devan Namputiri (1500- 1610) was a member of Parannottu family in Alathur Village. He was a student of Damodaran. Jyestha Devan was a younger contemporary and disciple of Nilakanda.

Jyestha devan is known as the author of Yukti Bhasha also known as Ganitha Nyaya Samgraham. This text is written in Malayalam. It explains the infinite series for Pi, Sine etc. The text contains two parts. The first part is devoted to the logical demonstration of mathematical research and the second part to astronomical topics. The sixth and seventh chapters of Yukti Bhasha contain proofs and derivations of the methods in the text. It gives derivations of π/16, π, π/8, etc., as rapidly converging series.

Yukti Bhasha also explains the methods to find the surface area of a sphere of radius r, as $4\pi r^2$ and volume of sphere as $\frac{4}{3}\pi r^3$.

Achutha Pisharady, 1550-1621, was the disciple of Jeyshtadevan. He wrote Uparaga Kriyakarmam in 1592. Melpathur Narayanabhattathiri who wrote Narayaneeyam was a disciple of Achutha Pisharody.

Sankara Varior

Sankara Varior (1500-1560) was a student of Neelakanda Somayaji and Damodaran. Karana Saram written in 1550 is the work of Chithrabhanu, which deals with astronomy. He also wrote several commentaries.

Puthumana Somayaji

Puthumana Somayaji (1660-1740) wrote Karana Padhathi in 1732. Karana Padhadhi contains 213 verses divided into ten chapters. It deals with the results and methods explained in Yukti Bhasha and Kriya Karmakari.

Sankara Varma

Sankara Varma was the younger brother of the King Udaya Varma. Sadratnamala is the only known work by Sankara Varman. It contains 211 verses in six chapters. Sadratnamala is a handbook of mathematics and astronomy. The original text was in Sanskrit and Sankara Varman also wrote a Malayalm translation. Sadratnamala gave the value of pi correct to 17 decimal places.

Bringing to limelight

Charles Wish, a civil servant of British Government, visited Kerala as part of his official assignment during the early part of 19th century. He collected details regarding contributions of Kerala Mathematicians. He prepared an article and presented it in Royal Asiatic Society of Great Britian and Ireland on December 15, 1832, which was published in 1835.

CONCLUDING REMARKS

The social and cultural background nourished an intellectual base in Kerala. The geographical advantages also helped to keep its pace undisturbed. The Mathematicians and astronomers of Kerala produced independent works. These were pioneers in many areas such as trigonometry, infinite series, etc. Kerala agriculture was dependent on the monsoon and so climate prediction was a major requirement. The social culture of considering exact times for all rituals also created a demand for measuring time and forecast. These led to the development of astronomy and, thereby, mathematics in Kerala.

The knowledge might have been transmitted to other parts of the world through foreigners who came to Kerala for trading of spices, etc.

Many of the results disvovered by Kerala Mathematicians were known as the works of few Europenas. But in the recent past, a lot of research has been done to identify the contributions of Kerala mathematicians. The *Golden Crest of Peacock* and *The Passage to Infinity* written by Dr George Ghevarghese Joseph have helped establish the validity of a non-Eurocentric history of mathematics. Now slowly people started recognizing the contributions of Kerala Mathematicians as pioneer in the areas like Infinite series etc.

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