COMPOUND INTEREST MENTALLY

Kuldeep Singh

Introduction

Calculating compound interest is a topic that takes much time to compute, especially if we need to find the amount for multiple years. Conventional methods require paper and pen to achieve complex calculations, but, using a Vedic method, we can calculate compound interest mentally very easily. We will explore the method and the logic behind it.

Objectives

- Understand and learn how to calculate compound interest mentally using Vedic method.
- Compare it with conventional method.
- Learn and understand the pattern for calculating it mentally.

Requirements

Two things are needed for this method

- The practice of simple percentages mentally
- Memory of the binomial coefficient pattern

With these, compound interest will be able to be calculated directly.

Some examples to practise percentage mentally are set out below.

- 10% of 2000 = 200
- 10% of 300 = 30
- 20% of 40 = 8
- 10% of 4 = 0.4
- 30% of 120 = 36
- 40 % of 4 = 1.6

Pattern for Compound Interest

The pattern used is that of the binomial coefficients and can be achieved through successive powers of 11 using the *Verically and crosswise* sutra.

1 year	1×11	=	(1) (1)
2 years	11×11	=	(1) (2) (1)
3 years	121×11	=	(1) (3) (3) (1)
4 years	1331×11	=	(1) (4) (6) (4) (1)
5 years	14641×11	=	(1) (5) (10) (10) (5) (1)
6 years	5 years $\times$ 11	=	(1) (6) (15) (20) (15) (6) (1)

and so on...

To use this table the first column of 1s is discarded.

Finding the Compound Interest

Example 1 Find the compound interest for 2 years on Rs. 2000 @ 5% per annum.

Solution (Conventional method) = $P \times (1 + r/100)^n - P$

$= 2000 \times (1 + 5/100)^2 - 2000$

$= 2000 \times (21/20)^2 - 2000$

$= 2000 \times 441/400 - 2000$

$= 2205 - 2000$

$= 205$ (compound interest)

Solution (Vedic Method) $= 2 \times 100 + 1 \times 5 = 205$

Explanation: The second row of the table has 2 and 1 as the multipliers.

5% of 2000 is 100 and 5% of 100 is 5. These are multiplied by 2 and 1 and summed to give the total interest.

Example 2 Find compound interest for 3 years on Rs. 6000 at 7% per annum

Solution: (Conventional Method) = $6000 \times (1 + 7/100)^3 - 6000$

$= 6000 \times 1225043/1000000 - 6000$

$= 7350.258 - 6000$

$= 1350.258$

Vedic Method: $= 3 \times 420 + 3 \times 29.4 + 1 \times 2.058$

$= 1260 + 88.2 + 2.058$

$= 1350.258$

Explanation: 7% of 6000 is 420, 7% of 420 is 29.4 and 7% of 29.4 is 2.058.

The respective multipliers are found in the 3rd row as 3, 3 and 1.

Example 3 Find the interest for 4 years on Rs. 50000 at 5% compounded annually.

Solution: (Conventional method) $= 50000 \times (1+5/100)^4 - 50000$

$= 50000 \times (21/20)^4 - 50000$

$= 5 \times 194481/16 - 50000$

$= 60775.3125 - 50000$

$= 10775.3125$

Vedic Method: $= 4 \times 2500 + 6 \times 125 + 4 \times 6.25 + 1 \times 0.3125$

$= 10000 + 750 + 25 + 0.3125$

$= 10775.3125$

Example 4 Find the compound interest for 5 years on 4000 at 4% per annum.

Solution: (Conventional method) $= 4000 \times (1 + 4/100)^5 - 4000$

$= 4000 \times (26/25)^5 - 4000$

$= 32 \times 11881376 / 78125 - 4000$

$= 380204032 / 78125 - 4000$

$= 4866.6116096 – 4000$

$= 866.6116096$

Vedic Method $= 5 \times 160 + 10 \times 6.4 + 10 \times 0.256 + 5 \times 0.01024 + 1 \times 0.0004096$

$= 800 + 64 + 2.56 + 0.05120 + 0.0004096$

$= 866.6116096$

It is very simple and with little practice it can also be done mentally.

When we need significant digits only, we can easily ignore the decimal part with this method.

Explanation of Method

For the first year/term the amount of simple interest and compound interest are same. But from 2^{nd} year/term we will get Simple interest along with interest on previous interests. The sutra for this is *By one more than the one before*. We can represent it in the table below:

Amount 10000 at 10%

	R1	R2	R3	R4	R5	R6
1^{st} Year	1000					
2^{nd} Year	1000	100×1				
3^{rd} Year	1000	100×2	10×1			
4^{th} year	1000	100×3	10×3	1×1		
5^{th} Year	1000	100×4	10×6	1×4	0.1	
6^{th} Year	1000	100×5	10×10	1×10	0.1×5	0.01

Note: R1 is r% of the amount

R2 is r% of R1

R3 is r% of R2

R4 is r% of R3

R5 is r% of R4

R6 is r% of R5

Explanation:

1^{st} Year	1000	= 1000
2^{nd} Year	$1000 \times 2 + 100 \times 1$	= 2100
3^{rd} Year	$1000 \times 3 + 100 \times 3 + 10 \times 1$	= 3310
4^{th} year	$1000 \times 4 + 100 \times 6 + 10 \times 4 + 1 \times 1$	= 4641
5^{th} Year	$1000 \times 5 + 100 \times 10 + 10 \times 10 + 1 \times 5 + 0.1 \times 1$	= 6105.1
6^{th} Year	$1000\times 6 + 100\times 15 + 10\times 20 + 1\times 15 + 0.1\times 6 + 0.01 \times 1$	=7715.61

So we have the same table as shown previously.

Explanation using Binomial theorem

Compound Interest for 2 years = $A\ (1 + r/100)^2 - A$

$$= (A)(1)^2 + 2(A)(1)^1(r)^1 + 1(A)(r)^2 – A$$

$$= 2(A)(1)^1(r)^1 + 1(A)(r)^2$$

Compound Interest for 3 years = $A\ (1 + r/100)^3 - A$

$$= (A)(1)^3 + 3(A)(1)^2(r)^1 + 3(A)(1)^1(r)^2 + 1(A)(r)^3 – A$$

$$= 3(A)(1)^2(r)^1 + 3(A)(1)^1(r)^2 + 1(A)(r)^3$$

Compound Interest for 4 years = $A\ (1 + r/100)^4 - A$

$$= (A)(1)^4 + 4(A)(1)^3(r)^1 + 6(A)(1)^2(r)^2 + 4(A)(1)^1(r)^3 + 1(A)(r)^4 – A$$

$$= 4(A)(1)^3(r)^1 + 6(A)(1)^2(r)^2 + 4(A)(1)^1(r)^3 + 1(A)(r)^4$$

This continues in like manner.

The compound interest can be calculated for *n* number of years to any degree of accuracy.

Example 5 Calculate the compound interest on 2500 for 30 years at 2% p/a correct to the nearest whole number.

Solution: Calculating this amount will take a long time with the conventional method but with the Vedic method we can calculate it as follows:

$30 \times 50 \ + \ 435 \times 1 \ + \ 4060 \times 0.02 \ + 27405 \times 0.0004 + 142506 \times 0.000008$

$= 1500 + 435 + 81.2 + 10.9620 + 1.140018$

$= 2028.302018$

Since further terms will not affect the answer to the nearest whole number we can say that it is 2028.

Conclusion

Using the binomial coefficients to calculate compound interest renders these calculations easy and accessible to school-aged students who normally struggle with the computations involved. This Vedic method can help both teachers and students appreciate the simplicity and utility of mathematical pattern. Students who learn this method will have an easy route to answering questions intheir exaimnations.

References

[1] Bharati Krishna Tirthaji Maharaja, (1994). Vedic Mathematics. Delhi: Motilal Banarasidas

[2] Kenneth R Williams, (2006). Discover Vedic Mathematics. Delhi: Motilal Banarasida