FINDING CUBE ROOTS: NEPALI AND VEDIC METHODS

Jayanta Acharya

Introduction

Mathematics is a study of quantity, structure, space and change. More specifically it is the study of Arithmetic, Algebra, Geometry and Analysis. Many students find mathematics a difficult and boring subject. If we use different methods to solve the same problem, students may come to enjoy the subject. Vedic Mathematics is becoming increasingly popular as more and more people are introduced to the beautifully unified and easy Vedic methods.

Finding the cube root of any number is a very difficult task for students. It is the reciprocal procedure of calculating the cube. If 8 is the cube of 2, then the cube root of 8 is 2. If 27 is the cube of 3, then the cube root of 27 is 3 and so on.

Unitary Method

Masny arithmetic problems can be solved by the unitary method but for finding square, square root, cube and cube root we cannot use unitary method. Bhaskara says

वर्गं वर्गपदं घनं घनपदं संत्यज्य यद् गण्यते

तत् त्रैराशिकमेव भेदबहुलं नान्यत् ततो विद्यते।

एतद् यद् बहुदास्मदादिजडधीधीवृद्धिबुध्द्या बुधै

विव्दच्चक्रचकोरचारुमतिभिः पाटीति तन्निर्मितम्॥ (शिध्दान्तशिरोमणि प्रश्नाध्याय)

(*Whatever we say about square, square root, cube and cube root, expect them the patiganitam is nothing but different forms of unitary method.*)

The unitary method is a technique which used for solving a problem by finding the value of a single unit and then finding the necessary value by multiplication of the single unit value.

Example

   If the cost of 5 liters of oil is Rs 25 then find the cost of 80 liter of oil. For such question we use unitary method

the cost of 5 liters of oil is Rs 25

the cost of 1 liter of oil is Rs $\dfrac{25}{5} = 5$

the cost of 80 liters of oil is Rs $\frac{25}{5} \times 80 = 400$

But for the case of cube this does not work.

If the cube of 2 is 8 then find the cube of 4?

The cube of 2 is 8, then, by proportion, the cube of 1 would be $\frac{8}{2} = 4$ in which case the cube would be $\frac{8}{2} \times 4 = 16$.

The same is true for finding a cube root inasmuch as simple proportion using the untary method does not work.

In this context, we discuss the method introduced by a Nepali scholar Gopal Pandey. He claims that the cube root can also be calculated by unitary method.

Gopal Pandey was a popular scholar of ancient mathematics in his commentary. His praise in local language Nepali can be seen in this form,

गोर्खाका मणिरामका सुत बडा यात्री बलीराम् भया

तिन्का पुत्रः हितै स्वदेशीजनका गन्र्या त गोपाल् भया ।

प्राचीन् सब् कडि खोल् दिन्या अति असल् ग्रन्थै बनाइदिया

तिन् ले व्यक्तगणित् सहज् गरिदिया सज्जनहरू खुस् भया ॥

Maniram's grandson and Baliram's son, Gopal Pandey, was from Gorkha and he wrote a manual of Arithmetic in Nepali. Bakyatya Chandrika (1895 AD) in Nepali and later on translated into Hindi.

His method is very useful and similar to Vedic method. His method is explained by examples in the following paragraphs.

Example: What is the cube root of 1728?

Place dots above the first digit on the right and then above every fourth digit. For 1728 the dots should be above 8 and 1.

$$\begin{array}{r} \dot{1}72\dot{8} \\ - \ 1000 \\ \hline 728 \end{array}$$

If there are two dots the answer is of two digits and, by observing, we can guess that the answer must be between 10 to 20, because the cube of 10 is 1000 and the cube of 20 is 8000. 1728 is between 1000 to 8000 so the answer is between 10 and 20. Pandey says since the difference of two cubes is (8000 - 1000 =) 7000, the difference of cube root is $(20 - 10 =)$ 10 .

Find the difference of the given number, 1728, and the cube of 10 (i.e. 1728 - 1000 = 728)

Now Pandey says use the unitary method like this,

Since the difference in two cubes is 7000 then the difference of two cube roots is 10.

If the difference in two cubes is 1 then the difference of two cube roots is 10/7000

If the difference in two cubes is 728 then the difference of two cube roots is $1\dfrac{28}{700}$

This indicates that the next digit must be greater than 1 and it should be less than 2. Pandey says the last digit must be 2 and the final answer must be 12.

Vedic Method

Example: Find the cube root of 2197

First make a group of three digits from the right 2 | 197

The cube ends in 7 and so the cube root will end with a 3. We will put 3 as the right hand part of the answer. For the first part of the answer look at the second part of the separation of given number there is 2 which lies between cube of 1 and 2 i.e. 1 and 8. The smaller number is 1, which we will put as the left hand part of the answer. Final answer is 13.

Example: Find the cube root of 1157625

Separate off three digits from the right, 1157 | 625. Since 1157625 ends with a 5 so the cube root also ends with a 5. The number to the left of the separation, 1157, lies between the cube of 10 (1000) and the cube of 11 (1331).

Out of 10 and 11, we take the smaller number 10 and put it beside the 5 already obtained. The final answer is 105.

The same problem is solved by using Nepali Unitary method (Pandey method) as follows:

Place the dot from the last digit of the given number and repeat it leaving two digits. For 1157625 the dot should be above 5, 7 and 1 so the cube root of the given number is of three digits.

$$\dot{1}\ 1\ 5\ \dot{7}\ 6\ 2\ \dot{5}$$
$$\underline{1\hphantom{\ 1\ 5\ 7\ 6\ 2\ 5}}$$
$$1\ 5\ 7\ 6\ 2\ 5$$

The answer must be between 100 to 110 because the cube of 100 is 1000000 and the cube of 110 is 1331000. Pandey says, if the difference of two cubes is 1000000 - 1331000 = 331000, the difference of cube roots is 110 - 100 = 10.

Find the difference of the given number and the cube of 100 (i.e. 1157625 -1000000 = 157625)

Now Pandey uses the unitary method like this,

If the difference in two cubes is 331000 then the difference of two cube roots is 10.

If the difference in two cubes is 1 then the difference of two cube roots is 10/331000.

If the difference in two cubes is 157625 then the difference of two cube roots is $4\dfrac{25225}{33100}$

This indicates that the next digit must be greater than 4 and it should be less than 5. Pandey says the last digit must be 5 and the final answer must be three digital so the answer is 105.

Here, we discussed how to find the cube roots of exact cube numbers comparing two different methods. Both scholars have suggested their own method to find the cube roots of non-exact cubes as well.

References

1. Jagadguru Swami shree bharati krisna tirthajee Maharaja, Sankaracharya of Govardahana Matha, Puri: *Vedic Mathematics,* Motilal Banarsidass, Reprint:Delhi, 1970, 1971, 1975, 1978, 1981, 1982

2. Bathia, Dhaval: *Vedic Mathematics Made Easy,* Jaico Publishing House, Second Jaico Impression: 2005

3. Gupta, Atul: *The Power of Vedic Maths,* Jaico Publishing House, First Jaico Impression: 2004

4. Kumar, Pradeep: *Vedic Mathematics*, SHREE Book Centre: Reprint 2003, 2004.

5. Glover, James T.: *Vedic Mathematics for Schools book 1, 2, 3*, Motilal Banarsidass Publishers Private   limited: 2002, 2003

6. Williams, Kenneth and Gakell, Mark: *The Cosmic Calculator 1, 2, 3,* Motilal Banarsidass Publishers   Private limited: First Indian Edition: Delhi, 2002.

7. Kapoor, Dr. S. K: *Learn and Teach Vedic Mathematics*, Lotus Press: 2004

8. Prakash, Vijay: *Speed Mathematics*, Vasan Publications: 2006, 2007

9. Hebbar, M. Satyamoorti, *Vedic Mathematics for Speed Arithmetic*,Vasan Publiations: 2006

10. Bhaskaracharya, *Lilavati*, With vasana of Pt. Damodara Mishra, Mithila Institut of Post Graduate studies   and research in Sanskrit Learning Darbhanga: 1959

11. नयराज पन्त, पण्डित गोपाल पाडे र उनका घनमूल ल्याउने रीति, नेपाल राजकीय प्रज्ञा प्रतिष्ठान, २०३७ वि.सं.

12. भाष्कराचार्य, शिध्दान्तशिरोमणि, जयदास हरिदासगुप्तः चौखम्बा संस्कृत सीरिज अफिस, विद्याविलास प्रेस वनारस, 1986

13. गोपाल पाडे, व्यक्तचन्द्रिका, निर्णयसागर, १९४० वि.सं