COMPARING CONVENTIONAL ITERATIVE METHODS TO THE VEDIC METHOD FOR DETERMINING ROOTS OF CUBIC AND QUARTIC EQUATIONS

Rick Blum

Abstract

Determining the roots of polynomial equations is fundamental with iterative methods available to do the tedious calculations. These iterative methods include the Bisection Method, the Regula-Falsi Method, the Newton-Raphson Method, the Secant Method and Halley's Method to name a few. The purpose of this paper is to compare how effective these methods are when compared to the Vedic Method. This comparison will be made on the basis of the number of calculations necessary to produce real roots to 4 decimal places for both simple and more complicated Cubic and Quartic equations using the above stated methods and comparing these results to that of the Vedic approach.

Introduction

The solution to cubic and quartic polynomial equations is notoriously difficult to determine using a formula approach. Determining roots for the quadratic equation in the form $ax^2 + bx + c = 0$ is relatively easy using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

If the roots of a polynomial are either integral or simple rational (p/q where both p and q are both integers), there are techniques available to determine the roots. Three of these techniques to be used in combination are:

1. Descartes' rule of signs, first described by Rene Descartes in his work La Geometrie, is a technique for determining an upper bound on the number of positive or negative real roots of a polynomial. It is not a complete criterion, because it does not provide the exact number of positive or negative roots. The rule is applied by counting the number of sign changes in the sequence formed by the polynomial's coefficients. If a coefficient is zero, that term is simply omitted from the sequence.

2. The Rational Root Theorem states a constraint on rational solutions of a polynomial equation with integer coefficients. These solutions are the "possible" roots (equivalently, zeroes) of the polynomial: $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ for some $a_0, ..., a_n \in \mathbf{Z}$.

If a_0 and a_n are nonzero, then each rational solution x, when written as a fraction $x = p/q$ in lowest terms (i.e., the greatest common divisor of p and q is 1), satisfies,

p is an integer factor of the constant term a_0, and

q is an integer factor of the leading coefficient a_n.

3. **Synthetic Division** – Once a potential root, "r," is determined using the above two techniques, synthetic division can be utilized to determine if dividing the given polynomial by the factor (x-r) will result in a 0 remainder.

The problem that develops is when the root(s) of the given polynomial are real, but, not integral.

One approach developed to determine cubic roots is a formula discovered by Girolamo Cardano in the 16th century to determine the solution of $ax^3 + bx^2 + cx + d = 0$ is

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$

$$+ \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}.$$

Or, more briefly,

$$x = \{q + [q^2 + (r-p^2)^3]^{1/2}\}^{1/3} + \{q - [q^2 + (r-p^2)^3]^{1/2}\}^{1/3} + p$$

where,

$$p = -b/(3a), \quad q = p^3 + (bc-3ad)/(6a^2), \quad r = c/(3a)$$

This formula will determine a root of the cubic equation with accuracy, but, on the other hand, to determine roots using this approach would be extremely time consuming and difficult. In addition, the formula to determine roots for quartic equations discovered by Lodovico Ferrari, a student of Cardano, is significantly more tedious and complicated.

In order to determine roots for cubic and quartic equations in a more efficient and simpler manner, iterative methods were developed which enables the expedious manual and/or computer solution of these types of equations. In this paper, I will calculate the roots of the following equations to 4 decimal places using iterative methods:

Cubic Equations:

$$x^3 - 100 = 0 \text{ where } x = 4.6416$$

$$x^3 + x^2 + 3x - 186 = 0 \text{ where } x = 5.2290$$

Quartic Equations:

$$x^4 - 100 = 0 \text{ where } x = 3.1623$$

$$x^4 - 2x^3 + 4x^2 - 5x - 10 = 0 \text{ where } x = 2.1810$$

I will provide a short description of each of the following methods followed by excel spreadsheets which have solved for the roots of the four above detailed equations to 4 decimal places: the Bisection Method, the Regula-Falsi (False Position) Method, the Newton-Raphson Method, the Secant Method and Halley's Method. I will indicate the number of arithmetic operations required to solve each equation which is a main indicator of speed of convergence. Finally, I will solve each of these equations using the Vedic approach and make a comparison of results.

Explanation of Iterative Methods

The Bisection Method

The Bisection Method is one of the oldest and most reliable ways to calculate the roots of a continuous function. It makes use of the Intermediate Value Theorem which basically states that: if f(x) is real and continuous over an interval $[x_L, x_U]$ and $f(x_L)f(x_U) < 0$, than there exists at least one real root between x_L and x_U.

The steps to use the Bisection Method to determine the real root of f(x) =0 are as follows:

1. Chose x_L and x_U as initial guesses. These guesses are not arbitrary. They must bracket the real root of the function to be determined. That is to say, $f(x_L)\ f(x_U) < 0$.

2. Determine the mid-point: $x_M = (x_L + x_U)/2$

3. Find whether $f(x_L)f(x_M)$ is <0, >0 or =0.

a. if $f(x_L)f(x_M) < 0$, then $x_L = x_L$ and $x_U = x_M$

b. if $f(x_L)f(x_M) > 0$, then $x_L = x_M$ and $x_U = x_U$

c. if $f(x_L)f(x_M) = 0$, then x_M is the root to be determined

4. Find new $x_M = (x_L + x_U)/2$

5. Determine $|\varepsilon| = |(x_{M(new)} - x_{M(old)}) / x_{M(new)}|$, the relative error of the estimate and check to see if this error is below a pre-specified tolerance.

6. If the relative error is less than the pre-specified tolerance, you are done. If not, then go back to step 2.

Advantages:

1. Always convergent

2. The relative error can be controlled and determined

Disadvantages:

1. Convergence is slow

2. The initial estimates need to bracket the root

3. Choosing an initial guess too close to a root may result in needing many iterations to converge

4. Cannot find roots for some functions that don't change sign over any interval. For instance, a

parabola $f(x) = x^2 + 1$ is always > 0.

5. This method could seek a singularity point as a root.

When utilized correctly after each iteration the interval containing the real root will be decreased by one half of its size. This is continued until the maximum defined error is obtained.

The Regula-Falsi Method

The Regula Falsi Method, also known as the False Position Method, was developed because the Bisection Method converges at a fairly slow rate. As before, we assume that $f(x_0)$ and $f(x_1)$ have opposite signs. As an aside, it was found that when the two initial guesses do not bracket the root, convergence will still take place, depending upon the function being evaluated, even though several hundred iterations may be required. The Bisection Method uses the midpoint of the interval $[x_0, x_1]$ as the next estimate. A better approximation is obtained if we find the point $(x_2, 0)$ where the secant line joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ crosses the x-axis. That is, a linear interpolation is performed between x_1 and x_0 to find the approximate root. To derive the iterative formula, write down two versions of the slope of the line:

$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

where the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ are used and

$$m = \frac{0 - f(x_1)}{x_2 - x_1}$$

where the points $(x_2, 0)$ and $(x_1, f(x_1))$ are used. Solving for x_2, we get the recursive formula:

$$x_2 = x_1 - \frac{f(x_1)\left(x_1 - x_0\right)}{f(x_1) - f(x_0)}$$

or, in general,

$$x_{n+1} = x_n - \frac{f(x_n)\left(x_n - x_{n-1}\right)}{f(x_n) - f(x_{n-1})}$$

To use the Regula Falsi Method:

1. Obtain two starting guesses, x_n and x_{n-1} such that $f(x_n)f(x_{n-1}) < 0$.

2. Produce the next iterate x_{n+1} from the formula above

3. Determine $|\varepsilon| = |(x_{M\,(new)} - x_{M\,(old)}) / x_{M\,(new)}|$, the relative error of the estimate and check

to see if this error is below a pre-specified tolerance. If required tolerance or maximum iterates have been reached then stop.

4. Otherwise continue as follows:

a) If $f(x_{n+1})$ has the same sign as $f(x_{n-1})$ redo step 2 above by assigning $x_{n-1} = x_{n+1}$.

b) If $f(x_{n+1})$ has the same sign as $f(x_n)$ redo step 2 above by assigning $x_n = x_{n+1}$.

Advantages:

1. Most times the Regula Falsi Method will converge faster than the Bisection Method.
2. Only one function is needed to perform the iterations, i.e., no derivatives of the function are needed as in both the Newton-Raphson and Halley's Methods.

Disadvantages:

1. May converge slowly for functions with big curvatures.
2. Newton-Raphson may be still faster if we can apply it.

The Secant Method

The Secant Method is slightly different than the Bisection Method. It takes two initial guesses between which there is a change in sign, but, instead of bisecting the difference between them after each iteration, it takes the values of the function at the initial points and constructs a secant line connecting the two. Where this line crosses the x-axis is the next guess and will replace this initial value with the same sign and repeat the process. This method can be summarized in the following way: let x_{i-1} and x_i be the initial guesses and let x_{i+1} be the next guess determined by the method:

$$x_{i+1} = x_i - \frac{f(x_i)\left(x_i - x_{i-1}\right)}{f(x_i) - f(x_{i-1})}$$

Using the Secant Method, we determine the root of $f(x) = 0$ as follows:

1. Initialize $i = 0$
2. Start with guesses x_1 and x_0
3. Use the formula above
4. Find $|\varepsilon| = |(x_1 - x_0) / x_1|$, the relative error of the estimate and check to see if this error is below a pre-specified tolerance.

5. If the relative error is less than the pre-specified tolerance, you are done. If not, then go back to step 3 and determine:

$$x_2 = x_1 - \frac{f(x_1)\left(x_1 - x_0\right)}{f(x_1) - f(x_0)}$$

Advantages:

1. This method converges faster than Bisection Method
2. It does not require the use of the derivative of the function
3. The two initial guesses do not need to bracket the root
4. It requires the evaluation of only function as compared to the NRM requiring two, the function and its derivative

Disadvantages

1. It may not converge
2. There is no guaranteed error bound for the computed iterates.
3. It is likely to have difficulty if $f'(\alpha) = 0$. This means the x-axis is tangent to the graph of $y = f(x)$ at $x = \alpha$.

Note: Both the Regula-Falsi and Secant Methods use a similar approach.in determining roots. The difference between the two approaches is what each method does with the each subsequent iterant once it′s determined. In the Regula-Falsi Method, we refine our range so that the root is always bracketed, as with the Bisection Method. In the Secant Method, we use the previous estimate plugged into the iteration formula to generate the value of the next iteration.

The Newton-Raphson Method

This method of determining the roots of a polynomial equation, $f(x) = 0$, has certain advantages over the Bisection Method. Whereas the initial guesses used to initiate the Bisection Method requires the guesses to not only bracket the root to be determined, but, its effectiveness wanes if the guesses are either too far or too close the root. In addition, as previously noted, the speed of convergence of the Bisection Method is slow. The Newton-Raphson Method (NRM) requires only one guess and this guess has basically no restraints on it with regard to where it is located relative to the root to be determined. In addition and very importantly, the rate of convergence of the NRM is quadratic rather than linear. One potential drawback is that this method involves determining the derivative of the function.

The NRM works like this: Let "a" be the initial guess and let "b" be the better guess. By NRM, $b = a - (f(a)/f'(a))$. Each successive iteration should bring us closer to the root of the function assuming this method produces convergence. Because Newton's Method has an evaluation with a derivative in the denominator, guesses close to where the derivative is equal to zero will not converge.

Using the NRM, we find the root of the equation $f(x) = 0$ as follows:

1. Given $f(x)$, determine the first derivative $f'(x)$.
2. Choose an initial guess x_0
3. Calculate $x_1 = x_0 - (f(x_0)/f'(x_0)$

4. Determine $|\varepsilon| = |(x_1 - x_0) / x_1|$, the relative error of the estimate and check to see if this error is below a pre-specified tolerance.
5. If the relative error is less than the pre-specified tolerance, you are done. If not, then go back to step 3 and determine $x_2 = x_1 - (f(x_1)/f'(x_1))$.

Advantages:

1. The method converges very quickly when it converges
2. Only requires one initial input to start the method
3. Initial guess is not restricted

Disadvantages:

1. Method may not converge
2. Need to evaluate two functions
3. The first derivative of the function needs to be determined

Halley's Method

Finally, for the sake of completeness, I will mention Halley's Method. Edmond Halley (1656-1742) discovered this method to determine the root of a continuous and differentiable polynomial function which has cubic convergence. Comparing this method to the others already discussed, the rate of converge is extremely fast. The iterative formula for this method is:

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2[f'(x_n)]^2 - f(x_n)f''(x_n)}$$

An interesting point needs to be made about the derivation of this method. When Halley wrote his paper describing this method in 1694, he apparently did not realize that his method involved derivatives or "fluxions" as he would have called them.

Advantages:

1. The major advantage of this method is its speed of convergence
2. It only requires 1 estimate and this estimate does not need to be close to the actual root

Disadvantages:

1. A disadvantage is that both 1st and 2nd derivatives of the function need to be determined before the method could be applied.
2. If the derivatives are equal to zero, the method will fail.

Explanation and Application of the Vedic Method

This method relies on Tirthaji's method of Flag division or has he referred to it "The Crowning Gem" of Vedic Mathematics, the use of duplexes, triplexes and quadruplexes and the application of the "Vertically and Crosswise" technique. I will detail each step in the solution of $x^3 - 100 = 0$. After this detailed solution, I will present the summary of the solutions for each of the 3 remaining equations I am solving. You will notice that for each significant figure in the answer, there will be at most 3 sets of calculations necessary.

Solution of $x^3 - 100 = 0$

Step 1: Estimate the answer to this equation. It appears that $x = 4$ provides a good approximation to the answer. This value will become our value "a" to be used subsequently.

Step 2: Determine the "multipliers" to be used in the solution of a cubic equation and plug-in the value "a." At this point, a definition of "multiplier" is needed:

Multipliers – the determination of "multipliers" is required in order to proceed with this approach. To develop the requisite multipliers, we apply the formula:

$f_n = f^n(x)/n!$, where:

f_n is the n^{th} multiplier

$f^n(x)$ is the n^{th} derivative of the equation to be solved

$n!$ is n factorial

Once these multipliers are determined, the initial estimate "a" is substituted in. For the function $f(x) = x^2 - 100$, the multipliers would be:

$f_1 = 3x^2/1! \rightarrow 3(a^2)/1!$

$f_2 = 6x/2! \rightarrow 6(a)/2!$

$f_3 = 6/3! \rightarrow 1$

Given that our initial estimate for "a" is 4, the multipliers will be:

1st multiplier is $3a^2 = 3(4^2) = 48$

2nd multiplier is $3a = 3(4) = 12$

3rd multiplier is $1 = 1$

Step 3: Set-up the solution as follows:

	100	0	0	0	0	0
48		12	1	0	0	0
	4 .					

Step 4: Subtract 4^3 from 100, which is 100 – 64 = 36 and put the result alongside the first entry to the right of the bar. The solution will look like this:

	100	|	$_{36}0$	0	0	0	0
48		|	12	1	0	0	0
	4	. |					

Step 5: Divide the multiplier, 48, into 360 and put down the result and remainder:

	100	|	$_{36}0$	$_{72}0$	0	0	0
48		|	12	1	0	0	0
	4	. |	6				

You will notice that 360 ÷ 48 = 7 with a remainder of 24. I indicated a quotient of 6 with a remainder of 72. The reason for this will become apparent in future steps as we try to avoid negative numbers.

Step 6: From this point forward, we will have a repeating sequence of steps: multiplication, subtraction and finally division. There is another set of concepts that need to be defined before we can proceed. The multiplication step from this point forward will involve the calculation of duplexes, triplexes and quadruplexes. I will denote these amounts by either a D, T, Q with a numerical subscript. These definitions are as follows:

$D_1(a) = a^2$

$D_2(ab) = 2(a)(b)$

$D_3(abc) = 2(a)(c) + b^2$

$T_1(a) = a^3$

$T_2(ab) = 3a^2b$

$T_3(abc) = 3(a)(b^2) + 3(a^2)(c)$

$Q_1(a) = a^4$

$Q_2(ab) = 4(a^3)(b)$

$Q_3(abc) = 6(a^2)(b^2) + 6(a^3)(c)$

For the derivation of these values, I would refer you to Kenneth Williams' book entitled "The Crowning Gem" published in 2013.

a) First we multiply the second multiplier, 12, by the duplex of 6, i.e. $D_1(6)$. The duplex of 6 is $6^2 = 36$. Therefore, our product will be (12)(36) = 432.

b) Subtract this result from our prior result of 720, which is 288.

c) Divide this last result of 288 by 48. Again, even though the result is 6 with no remainder, we choose to use a quotient of 4 with a remainder of 96. Our result will now look like this:

	100	$_{36}0$	$_{72}0$	$_{96}0$	0	0
48		12	1	0	0	0
	4 .	6	4			

You will notice that the amount to be subtracted tends to grow and this is the reason to generate a large remainder so that when the subtraction step occurs, the result will be positive.

Now the cycle of multiply, subtract and divide is repeated with another decimal place being generated each time.

Step 7:

a) We now multiply the second multiplier, 12, by the duplex of 64, i.e. D2(64). The duplex of 64 is (2)(6)(4) = 48. Therefore our product here will be (12)(48) which is 576 and add to it the product of 1 and the triplex of 6, i.e. $T_1(6)$. The triplex of 6 is 6^3 which is 216. The sum of these products will be 576 + 216 = 792.

b) Subtract this result from our prior result of 960 which is 168.

c) Divide this last result of 168 by 48. Again, even though the result is 3 with a remainder of 24, we choose to use a quotient of 1 with a remainder of 120. Our result will now look like this:

	100	$_{36}0$	$_{72}0$	$_{96}0$	$_{120}0$	0
48		12	1	0	0	0
	4 .	6	4	1		

Step 8:

a) We now multiply the second multiplier, 12, by the duplex of 641. The duplex of 641 is $(2)(6)(1)+4^2$ = 28. Therefore our product here will be (12)(28) which is 336 and add to it the product of 1 and the triplex of 64. The triplex of 61 is $(3)(6^2)(4)$ which is 432. Finally, multiply the 4^{th} multiplier, which is 0, by the first quadruplex of 6, $Q_1(6)$. This quadruplex is equal to 6^4 which equals 1,296. The sum of these products will be 336 + 432 + 0 = 768.

b) Subtract this result from our prior result of 1200 which is 432.

c) Divide this last result of 432 by 48. Again, even though the result is 9 with no remainder, we choose to use a quotient of 6 with a remainder of 144 to avoid future negative subtractions.

Our result will now look like this:

	100	$_{36}0$	$_{72}0$	$_{96}0$	$_{120}0$	$_{144}0$
48		12	1	0	0	0
	4 .	6	4	1	6	

At this point, we have a solution to 4 decimal places.

The solution to the remaining 3 equations are as follows:

Solution to $x^4 - 100 = 0$

	100	$_{19}0$	$_{82}0$	$_{118}0$	$_{304}0$	$_{339}0$
108		54	12	1	0	0

	3 .|	1	6	2	3	

Solution to $x^3 + x^2 + 3x - 186 = 0$

	186 |	$_{21}0$	$_{34}0$	$_{100}0$	$_{72}0$	$_{56}0$
88	|	16	1	0	0	0
	5 .|	2	2	9	0	$\bar{2}$
	5 .	2	2	9	0	

For the sake of illustration, I will show the calculations of the multipliers, duplexes, triplexes and quadruplexes used in this solution:

Assuming that our estimate of the root is 5, the multipliers will be:

$f_1 = (3(a^2) + 2(a) + 3)/1! \rightarrow 3(5^2) + 2(5) + 3 = 88$

$f_2 = (6(a) + 2)/2! \rightarrow 3(5) + 1 = 16$

$f_3 = 6/3! = 1$

$D_1(2) = 2^2 = 4$

$D_2(22) = (2)(2)(2) = 8$

$D_3(229) = (2)(2)(9) + 2^2 = 40$

$T_1(2) = 2^3 = 8$

$T_2(22) = 3(2^2)(2) = 24$

$T_3(229) = 3(2)(2^2) + 3(2^2)(9) = 132$

$Q_1(2) = 2^4 = 16$

Solution to $x^4 - 2x^3 + 4x^2 - 5x - 10 = 0$

	10 |	$_{4}0$	$_{2}0$	$_{\bar{6}}0$	$_{1}0$	$_{10}0$	
19	|	16	6	1	0	0	
	2 .|	2	$\bar{2}$	1	0	2	.
	2	1	8	1	0		

Comparing Number of Arithmetic Operations to Produce Final Results

Bisection Method	Number of Arithmetic Operations	% of Conventional Method*
$f(x) = x^3 - 100$	246	10.57%
$f(x) = x^4 - 100$	309	10.03%
$f(x) = x^3 + x^2 + 3x - 186$	390	8.72%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	579	5.18%
Regula-Falsi Method		
$f(x) = x^3 - 100$	210	12.38%
$f(x) = x^4 - 100$	763	4.06%
$f(x) = x^3 + x^2 + 3x - 186$	210	16.19%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	1194	2.51%
Secant Method		
$f(x) = x^3 - 100$	80	32.50%
$f(x) = x^4 - 100$	96	32.29%
$f(x) = x^3 + x^2 + 3x - 186$	96	35.42%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	240	12.50%
Newton-Raphson Method		
$f(x) = x^3 - 100$	39	66.67%
$f(x) = x^4 - 100$	39	79.49%
$f(x) = x^3 + x^2 + 3x - 186$	38	89.47%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	57	52.63%
Halley's Method		
$f(x) = x^3 - 100$	42	61.90%
$f(x) = x^4 - 100$	42	73.81%
$f(x) = x^3 + x^2 + 3x - 186$	54	62.96%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	70	42.86%

Vedic Method

$f(x) = x^3 - 100$	26	100.00%
$f(x) = x^4 - 100$	31	100.00%
$f(x) = x^3 + x^2 + 3x - 186$	34	100.00%
$f(x) = x^4 - 2x^3 + 4x^2 - 5x - 10$	30	100.00%

* This column compares the number of steps of the Vedic Method number of steps of the traditional methods. For example, I compared the number of steps to solve $f(x) = x^3 - 100$ for the Vedic Method, 26 steps, to the number of steps for the Bisection Method, 246 steps. That is 26/246 = 10.57%

Conclusions

The purpose of this analysis was to determine if the solution of cubic and quartic equations would require less arithmetic operations using Vedic methods as compared to conventional iterative approaches.

As indicated earlier in this paper, there are formula-based methods available to help solve cubic and quartic equations. Unfortunately, once the equation attains 3rd degree or higher, these formula-based approaches become complicated and very cumbersome. There are methods, i.e. Descartes Rule of Signs, the Rational Root Theorem and Synthetic Division explained earlier, that can assist in root determination as long as the solution is fairly simple. Once the order of the equation equals 3 or above, these approaches are not the methods of choice. Over the years, mathematicians have developed iterative methods that can be used to solve more complicated equations. These methods are not only computationally complicated, but, are also too tedious to work out by hand. With the advent of computers, the time it takes to use these iterative methods to solve equations has been greatly reduced.

The five iterative methods that I examined in this paper can be divided into two categories: those requiring knowledge of calculus to apply and those that don't. The first group of methods includes the Bisection Method, the Regula-Falsi Method and the Secant Method. The 2nd group, requiring a knowledge of calculus, consists of the Newton-Raphson Method and Halley's Method.

For each of these methods, I developed solutions to 4 equations: two 3rd degree and two 4th degree equations of varying complexity. I then used excel spreadsheets to determine the number of iterations to determine the roots to 4 decimal places. Based upon these spreadsheets I was able to determine the number of arithmetic operations necessary to arrive at their respective solutions.

The Vedic Method was then used to determine the roots. I compared the number of steps that each method took to determine solutions for each of the equations to the Vedic Method and summarized them on the "Comparing Number of Arithmetic Operations to Produce Final Result" exhibit.

With regard to the Bisection and Regula-Falsi Methods, the simplicity of the methods were more than outweighed by the number of steps needed to arrive at solutions. Each method required hundreds of steps to attain the accuracy we wanted. In fact, the Regula-Falsi method required almost 1,200 arithmetic operations to arrive at the required accuracy when solving the more complicated 4^{th} degree equation.

The Secant Method did better than the previous methods requiring just fewer than 100 steps to solve the first 3 equations. In solving the more complicated 4^{th} degree equation, the number of steps jumped to 240.

In comparing the Newton-Raphson Method and Halley's Method, both approaches required about the same number of steps to arrive at their respective solutions. These methods produced results more effectively than the prior approaches. The number of steps required varied between 38 – 70 to arrive at the requisite solutions. Both of these solutions did require an elementary knowledge of calculus to differentiate the equations for use in the method.

The Vedic Method, as can be seen on the comparison, required **substantially** less steps to determine roots with the required accuracy. In fact, the Vedic Method was from 2 to almost 40 times faster than the conventional methods.

I believe that the results shown in this paper demonstrate the power that the Vedic Method has over traditional mathematical approaches to determine roots of non-linear equations. Even with the advent of the immense computational power of computers, if the number of calculations performed to arrive at an acceptable solution in a given problem is reduced dramatically by the use of Vedic methodologies, then the Vedic approach should be utilized.

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