PRACTICAL MATHEMATICS

Kenneth Williams, Vedic Mathematics Academy, U.K.

Abstract

This paper shows that some common mathematical procedures can be simplified by taking one further but significant step. This leads not just to simple patterns and easier maths in a practical sense but also to unexpected connections that link different mathematical topics. After giving some known Vedic mathematics examples of this, the paper applies the idea to finding the equation of a line or a perpendicular bisector, through two given points, and the equation of a circle or parabola, though three given points, and these four procedures are found to have connected structures.

1. Introduction

One of the most attractive features of the Vedic system of mathematics is the simplicity of the methods, the aim being to go straight to the answer by the shortest route. However, we find that what is commonly taught in schools is often a roundabout way to arrive at that answer; the child must routinely go through a procedure often a much longer one than is necessary.

It is of course fine for children to use the tools they have learned to explore new areas and become confident and competent in their application, but it is not necessary that having found a route to the solution of a problem they should always take that route in the future, especially when a simpler one is available.

This paper shows and discusses how commonly repeated procedures can be simplified to reveal short and satisfying patterns that are not only simple, memorable and highly efficient, but also interconnected.

We don't prove Pythagoras' Theorem every time we apply it; we see it as a result which we can take for granted and apply. We don't find 53×10 by using long multiplication because we know of a much more efficient method.

It is useful if the procedures we learn, like the two examples just quoted, can be assimilated as a unit and used as a tool which can then be applied to further work in mathematics. For this to happen most efficiently we need to have those procedures in their most efficient form.

After illustrating this point in Section 2 with specific examples, we go on to show that Tirthaji's striking method of finding the equation of a line joining two points can be extended to similar methods in coordinate geometry. We find that by simplifying techniques that produce specific results further than is normally done, we arrive at unifying patterns that naturally group a set of those techniques.

That is, the equation of a line through or perpendicular to two given points, or the equation of a circle or parabola through three given points, can be found easily, and further, that these four methods have unifying characteristics.

2. Known Vedic Examples

There are many illustrations that show the contrast between conventional methods and Vedic ones with regard to efficiency.

2.1 Subtraction

As a first example we may take subtraction from a base number (i.e. a power of 10 or multiple thereof). In finding say 10,000 – 6787 the student is expected to first set it out and then, starting at the right end try to subtract in the right-most column.

$$\begin{array}{r} 10000 \\ -\ \ 6787 \\ \hline \end{array}$$

Since this ‘cannot be done’ they go along to the first non-zero digit in the minuend. This leads to crossing the 1 out and replacing it by 0, and the 1 is then placed against the next digit in the minuend to create 10, which is then immediately crossed out and replaced by 9 so that a 1 can be carried over to the next place. And so on.

$$\begin{array}{cccccc} & {}^{0}\cancel{1} & {}^{1}\cancel{0}^{9} & {}^{1}\cancel{0}^{9} & {}^{1}\cancel{0}^{9} & {}^{1}0 \\ - & & 6 & 7 & 8 & 7 \\ \hline \end{array}$$

After all this we find we can subtract in each column to get 3213 as the difference.

And as can be seen what we do is take each of the first three digits in 6787 from 9 and the last one from 10.

Once it is appreciated that this “all from 9 and the last one from 10” will be applied every time that we subtract a number from its next highest base number (power of 10) why not assimilate this and apply “all from 9 and the last one from 10” every time, at the outset.

Since this type of subtraction is in frequent use in getting change in a shop, and in other situations, it seems absurd to not teach this simple rule (which also leads to a completely general subtraction method).

The current situation in conventional mathematics for such subtractions is not suited for everyday use. But we can see here that by taking that extra step towards efficiency we arrive quickly at a more practical mathematics.

2.2 Adding and Subtracting Fractions

Another prominent example is in adding and subtracting fractions.

If we need to find $\frac{2}{5}+\frac{3}{7}$ we need to first find the common denominator, which is 35.

$$\frac{2}{5}+\frac{3}{7}=\frac{\qquad}{35}$$

Then we go through a procedure: divide the first denominator into the common denominator, multiply the result by the numerator of the first fraction and put this down over the 35. Doing this also with the second fraction we get:

$$\frac{2}{5}+\frac{3}{7}=\frac{14+15}{35}=\frac{29}{35}.$$

This long and complex procedure can be simplified by just cross-multiplying and adding to get the numerator (2×7 + 3×5) and multiplying the denominators to get the denominator.

Since there is a simple pattern we can give the answer in one step, mentally.

It is also easily explained since we need only to multiply the top and bottom of each fraction by the other fraction's denominator to arrange for both fractions to have the same denominator.

It seems this method is actually taught in schools in Holland, though the variations that can be applied when the denominators are not relatively prime or when we have three or more fractions, is not taught.

This illustrates once again how a little further development can lead to a more child-friendly procedure.

2.3 Products Near a Base

Another illustration of this point is in finding the product of two numbers near a base. For example 87 × 97, which are both close to 100.

A student wondering if the fact that the numbers are close to 100 could be used to get the answer more efficiently might start by thinking of taking 3 87s from 100 87s:

$$87 \times 97 = 87\times100 - 87 \times 3.$$

And since 87 is 13 below 100 we could find 87×3 by finding 300 – 13×3.

$$\begin{aligned} \text{So } 87 \times 97 &= 8700 - 87\times3 \\ &= 8700 - 300 + 13\times3 \\ &= 8400 + 39 \\ &= 8439. \end{aligned}$$

Now since this method can be applied to products of all pairs of numbers near 100, we may simplify this further and arrive at the neat pattern where we take one deficiency from the other number and then multiply the deficiencies.

A visual thinker may arrive at the pattern:

$$\begin{array}{l} 87 - 13 \\ \underline{97 - \ \ 3} \\ \underline{84\ /\ 39} \end{array}$$

showing that we just cross-subtract (87–3 or 97–13) and multiply the deficiencies vertically.

This is a nice result which students can arrive at by themselves (possibly with some assistance) and which will enable them to find such products mentally and to extend the idea in various ways.

The point is that, even if a child uses the base of 100 to solve the problem of 87×97, the method itself may not then be condensed into a simple pattern that can be used on all products of a certain type. And that simple pattern is what we should be aiming at.

2.4 Solution of Equations

In Chapter 11 of his book[1] where Tirthaji is solving equations of the type $ax + b = cx + d$ he writes:

"$2x + 7 = x + 9 \therefore 2x - x = 9 - 7 \therefore x = 2$. The student has to perform hundreds of such transposition-operations in the course of his work; but he should by practice obtain such familiarity with and such mastery over it as to assimilate and assume the general form that if $ax + b = cx + d$, $x = \frac{d - b}{a - c}$ and apply it by mental arithmetic automatically to the particular case actually before him and say:

$2x + 7 = x + 9 \therefore x = \frac{9-7}{2-1} = \frac{2}{1} = 2$; and the whole process should be a short, and simple mental process."

Here Tirthaji is making the same point: that it is worth assimilating a pattern to avoid unnecessarily repeating a long process.

2.5 Equation of a Line Joining Two Given Points

There are two ways this is commonly taught.

1. by substituting the given values into the general equation for a straight line, $y = mx + c$, to get two equations, which are then solved simultaneously.

2. by substituting into the formula $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

Both these are long-winded and the student is expected to go through this long process every time.

Teachers are surprised when they see Tirthaji's method to get the equation of a line joining two points using a simple vertical and crosswise pattern. Yet it is easily obtained in a couple of simple steps from the second of the two equations above.

2.5.1 Example

Find the equation of the straight line joining the points (7,4) and (5,1)

We write the coordinates one below the other as shown below:

$$\begin{array}{ccccc} 7 & & 4 & & \\ 5 & & 1 & & \\ \hline (7-5)y & = & (4-1)x & + & 7\times1-4\times5 \\ \hline & 2y & = 3x-13 & & \end{array}$$

The answer is $2y = 3x - 13$ and this can be written straight down.

The coefficient of y is found by subtracting the x coordinates (7–5=2).

The coefficient of x is found by subtracting the y coordinates (4–1=3).

And the last term, –13, is found by cross-multiplying and subtracting: (7×1–4×5=–13).

Mathematics needs to be practical, so children need to end up with the most efficient and most memorable method for solving specific problems. Strangely enough the most efficient method seems to also be the most memorable. It may be necessary to have the children take a longer route in the beginning in order to establish understanding and the validity of some method but in the end they should be equipped with the best method – the 'best method' being the one that is preferred by the student out of all the methods practised.

3. New Examples

We may further illustrate how simple patterns can arise by taking some problems similar to the one just above: the equation of a line joining two given points.

Two points also define a line which is their perpendicular bisector: is it possible to write down this equation immediately by finding a simple pattern that can always be applied? And it would be interesting to see if such a pattern was similar to the one already found for the equation of a line joining the two points.

Further, since *three* given points can define a circle or a parabola, we will look for simple patterns which can be used to determine their equations.

3.1 Perpendicular Bisector of a Line Joining 2 Given Points

Usually we solve this by finding the coordinates of the mid-point of the line joining the points and also the gradient of that line. We then find the gradient of the perpendicular line, substitute this into $y = mx + c$ as m, and then substitute the coordinates of the mid-point to find the value of c.

It is a straightforward but somewhat fiddly process.

We can illustrate the Vedic method with an example.

3.1.1 Example

Find the equation of the perpendicular bisector of the line joining (3,8) and (1,4).

x	y	x^2+y^2
3	8	73
1	4	17
$2x + 4y$	=	28

The third, auxiliary, column is for the sum of the squares of the coordinates.
We subtract vertically in each of the three columns, but in the 3rd column we halve the difference (½(73–17) = 28) .

The answer is $2x + 4y = 28$ or $x + 2y = 14$.

This is very simple and a matter of mental arithmetic.[2]

Similarly, for (3,-1) and (-2, 5):

3	-1	10
-2	5	29

$5x - 6y = -9½$ or $10x - 12y = -19$

The vertical subtractions are similar to those used when finding the equation of the line joining the given points. The pattern of the result is different though: $ay = bx + c$ in the first case, and $ax + by = c$ in the second.

3.2 Equation of a Circle Through 3 Given Points

There are several standard ways to go about this, all of which involve a lot of calculation and algebra.

Alternatively we may proceed as illustrated in the example below.

3.2.1 Example

Find the equation of the circle that passes through the points (6,5), (5,2) and (4,1) in the form $(x - a)^2 + (y - b)^2 = r^2$.

We again place the points under each other and form the auxiliary column x^2+y^2.

x	y	x^2+y^2	
6	5	61	row 1
5	2	29	row 2
4	1	17	row 3
a +	$3b$ =	16	row 4
a +	b =	6	row 5

We next subtract two of the three rows.
If we start with row 1 minus row 2, we get $a + 3b = 16$, as shown. Note that once again we halve the differences in the 3rd column.

Next we choose two different rows to subtract.
If we take row 2 minus row 3, we get $a + b = 6$ as shown.

Now solving rows 4 and 5 simultaneously we easily find that $b = 5$ and $a = 1$.

This gives $(x - 1)^2 + (y - 5)^2 = r^2$. (1)

r^2 is found by substituting one of the given coordinates into the LHS of (1). If we take (4,1) we have $(4 - 1)^2 + (1 - 5)^2 = 9 + 16 = 25 = r^2$.

So the final answer is $(x - 1)^2 + (y - 5)^2 = 25$.

When choosing the rows to subtract we have two ways to choose them out of three options. Of course we choose the ones that make the solution of the resulting equations easiest.

Should the resulting simultaneous equations not be as simple as in the example above we can use the standard Vedic method[1] for solving such pairs of equations.

3.3 Equation of a Parabola Through 3 Given Points

Perhaps the simplest approach to this would be to start with $y = ax^2 + bx + c$, substitute three times into this and solve the resulting three equations simultaneously.

Doing this, and looking closely at the steps to see what will happen every time we go through such a solution, we find an easy way of getting the answer, as illustrated by the example below.

3.3.1 Example

Find the equation of the parabola that passes through the points (4,3), (2,1) and (1,3), in the form $y = ax^2 + bx + c$.

Once again we place the points under each other, and we form an auxiliary column, but this time it is for x^2, and it is best placed first.

x^2	x	y	
16	4	3	row 1
4	2	1	row 2
1	1	3	row 3
$12a + 2b = 2$			row 4
$15a + 3b = 0$			row 5

Again we subtract in the columns and form two out of three possible equations.

If we take row 1 minus row 2, and row 1 minus row 3, we get the equations shown above in rows 4 and 5.

Simplifying these we get:
$6a + b = 1$
$5a + b = 0$ which can then be easily solved to give $a = 1$ and $b = -5$.

Our parabola is therefore $y = x^2 - 5x + c$, and c is found by selecting any one of the three given points and finding $y - (x^2 - 5x)$. If we choose (1,3) we find $c = 3 - -4 = 7$.

So $y = x^2 - 5x + 7$.

3.4 We find in these last four methods that there are simple patterns that can be used to solve these problems easily. We note also that there are convenient similarities in them.

4. Concluding Remarks

Many examples have been offered here illustrating how simple patterns can be used to solve various problems. These patterns make solving problems easier because they make the approach to their solution more memorable and more satisfying.

It would appear that our modern methods sometimes do not go far enough to get the best technique to solve a particular problem: that we can arrive at neater results by simplifying a method as far as it will go.

It would be interesting and potentially very constructive to look for unified procedures in this way in other areas of mathematics.

We may describe mathematics as "the search for unity": whether that unity is the product of two numbers, the solution to a puzzle, a mathematical proof, etc. But the search for unity can also apply to the unification of related solution methods, which can be satisfying as well as of practical use.

References

[1] Bharati Krishna Tirthaji Maharaja, (1965). Vedic Mathematics. Delhi: Motilal Banarasidas,.

[2] Babajee D. K. R. & Williams K. R. (2013). Vedic Mathematics Proofs. U.K.: Inspiration Books