DIVISIBILITY CHECKS USING OSCULATION

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Abstract

A divisibility check involves determining whether a given number is divisible by a particular divisor without having to actually carry out a division. A process called "osculation" (which makes use of an "osculator") is a unique method from Vedic Mathematics which can be employed to check divisibility. The method is an application of the Vedic sub-sutra *Veshtanam* ("By Osculation"). There are two types of osculators, positive osculators and negative osculators. This paper discusses divisibility checks for divisors between 2 and 50, and also investigates the osculation of binary numbers.

Objectives

- A divisibility check for divisors ending with 9 involving positive osculation and the use of the *Ekadhikena* sutra.
- A divisibility check for divisors ending with 1 involving negative osculation and the use of the *Ekanyunena* sutra.
- A divisibility check for divisors ending with 3 or 7 involving positive and negative osculation.
- A divisibility check for divisors which are powers of 2 or 5 using the sutra *Seshanyankena Caramena* ("The remainders by the last digit").
- Osculation of binary numbers.

Division

Division is one of the four basic mathematical operations and is a basic skill used in daily life. Division involves distributing a group of items into equal parts. A divisibility check is done on a number to determine whether it can be divided by a divisor, without actually having to carry out the division. The sutras *By Osculation* and *The Remainders by the Last Digit* can be employed to check the divisibility of any number by a particular divisor.

Osculation

Osculation is the name of a process used to check the divisibility of a number by a divisor ending with 9 or 1. An "osculator" is the tool used in this process. The sutras *Ekadhikena* and *Ekanyunena* are used to help find the osculator (E) associated with a particular divisor.

Example How to find the osculator for 49

The sutra *Ekadhikena Purvena* means “By one more than the one before”. One more than 49 is **5**0, so the osculator is 5.

Example How to find the osculator for 71

The sutra *Ekanyunena Purvena* means “By one less than the one before”. One less than 71 is **7**0, so the osculator is 7.

Positive Osculation

Positive osculation is used to check the divisibility of a number by a divisor ending with 9. The osculator must first be found, and then the number is osculated from right to left.

The process of osculation involves multiplying the right-most digit of the number being investigated by the osculator, and then adding the product to the digit just preceding it. The answer thus obtained is written below this digit. If this answer comprises two digits, the tens digit is carried to the left, while the unit digit is again multiplied by the osculator, before being added to the digit preceding it. If a tens digit has been carried over to the left, this digit is also added. The answer is again written below the digit which has most recently been osculated. This process is repeated until the last digit has undergone osculation, and a number has been written below it. If this number is either zero or is, itself, divisible by the divisor (from which the osculator was obtained), then the number under investigation is confirmed to be divisible by that divisor. Otherwise, it is not.

Example Check the divisibility of 23983 by 29.

Using the *Ekadhikena* sutra, one more than 29 is 30, therefore the osculator $E = 3$.

Osculate from right to left.

2	3	9	8	3
9*3+2	1*3+3+3	7*3+1+9	3*3+8	
29	9	31	17	

The last number after osculation is 29, and 29 is divisible by itself, so the number 1074479 is indeed divisible by 29.

The table below shows the osculators for some divisors ending with 9.

Divisor	**Osculator (*E*)**
9	1
19	2
29	3
39	4
49	5

Note: Osculation using the positive osculator 1 always results in the digit sum of the number.

Negative Osculation

Negative osculation is used to check the divisibility of a number by a divisor ending with 1. When doing negative osculation, a bar is placed from right to left on alternate digits of the dividend. After the osculator has been determined, osculation is carried out from right to left using the same method discussed for positive osculation. Depending on whether a particular digit has a bar on it or not, either a subtraction or an addition is carried out in that particular step. All tens digits carried to the left must be subtracted.

Example Check the divisibility of 25543 by 41.

Using the *Ekanyunena* sutra, one less than 41 is 40, therefore the osculator $E = 4$.

Bars are placed on alternate digits of the dividend from right to left. Digits with bars on are considered to be negative. Osculate from right to left.

2	$\bar{5}$	5	$\bar{4}$	3
$0*4+\bar{2}+2$	$7*4+\bar{3}+\bar{5}$	$8*4+5$	$3*4+\bar{4}$	
0	20	37	8	

The last number after osculation is 0, which indicates that 25543 is divisible by 41.

The table below shows the osculators for some divisors ending with 1.

Divisor	**Osculator (E)**
11	1
21	2
31	3
41	4

Divisibility Check for Divisors Ending with 3 or 7

A divisor ending in a 3 or a 7 must first be converted to a below-base number or an above-base number by multiplying by either 7 or 3. (7*3 yields a product ending on 1, which results in the use of a negative osculator. 3*3, as well as 7*7, results in a product ending on 9, which yields a positive osculator.) After finding the osculator, positive or negative osculation is carried out to complete the divisibility check.

Example Check the divisibility of 4658 by 17.

Choosing positive osculation, 17 * 7 = 119.

Using the *Ekadhikena* sutra, one more than 119 is 120, therefore the osculator *E* = 12.

Do positive osculation from right to left.

4	6	5	8
8*12+2+4	1*12+10+6	8*12+5	
102	28	101	

The final number after osculation is 102 which is divisible by 17, so the number is divisible by 17.

If the final number after osculation is quite large, we can further osculate to make it smaller. For the case above, 102 can, itself, be osculated by 12 as shown below.

1	0	2
4*12+2+1	2*12+0	
51	24	

51 is divisible by 17.

We can osculate 51 even further:

5	1
1*12+5	
17	

17 is divisible by 17, so the number is divisible by 17.

Sometimes, when this process is repeated even further, the same answer is again eventually obtained - this indicates divisibility.

Example Check the divisibility of 4658 by 17.

Choosing negative osculation, 17 * 3 = 51.

Using the *Ekanyunena* sutra, one less than 51 is 50, therefore the osculator *E* = 5.

Proceed with negative osculation from right to left, after placing bars on alternate digits.

$\bar{4}$	6	$\bar{5}$	8
8*5+$\bar{2}$+$\bar{4}$	5*5+$\bar{3}$+6	8*5+$\bar{5}$	
34	28	35	

34 is divisible by 17, so the number is divisible by 17.

Divisibility Check for Divisors that are Powers of 2 or 5

To carry out a divisibility check for divisors that are powers of 2 or 5 (i.e. 2^n or 5^n), divide the number being investigated by 2 or 5. Divide the quotient thus obtained again by 2 or 5, and repeat this process a maximum of *n* times. If the final remainder is 0, then the number is divisible. Otherwise, it is not.

All even numbers are divisible by 2. To check the divisibility of a number by 4 (i.e. 2^2), divide the number twice by 2. If the remainder is 0 both times, the number is divisible by 4. To check the divisibility of a number by 8 (i.e. 2^3), divide the number by 2 three times. If the remainder is 0 each time, the number is divisible by 8, otherwise it is not. To check the divisibility of a number by16 (i.e. 2^4), divide the number four times by 2; if 32 (i.e. 2^5) is the divisor, divide the number five times by 2, etc. If the remainder is 0 every time, the number is divisible by that particular power of 2.

Example Check the divisibility of 22608 by 16.

$16 = 2^4$

Divide the number by 2 a maximum of four times.

Division	22608÷2	11304÷2	5652÷2	2826÷2
Quotient	11304	5652	2826	1413
Remainder	0	0	0	0

A remainder of 0 is obtained in each step, so 22608 is divisible by 16.

Note For a divisor 2^n, the process can, of course, be terminated in less than *n* steps, the moment a non-zero remainder is obtained.

Numbers ending in a 0 and a 5 are divisible by 5. To check the divisibility of a number by 25 (i.e. 5^2), divide the number twice by 5. If the remainder is 0 both times, the number is divisible by 25, otherwise it is not.

Example Check the divisibility of 6325 by 25.

$25 = 5^2$

Divide the number twice by 5 and check the remainder.

Division	6325÷5	1265÷5
Quotient	1265	253
Remainder	0	0

The remainder is 0 in both steps, so 6325 is divisible by 25.

Divisibility Check Using Factorisation

If the divisor does not end in a 9, 1, 3 or 7, and is also not a power of 2 or 5, then write the divisor as the product of two factors. Check whether the factors are divisors of the number being investigated for divisibility. If both factors are divisors, then their product is also a divisor.

Example Check the divisibility of 8132 by 38.

38 = 19 * 2

If 8132 satisfies divisibility checks for both 2 and 19, then it is divisible by 38.

8132 is an even number, so it is divisible by 2.

To check for divisibility by 19, positive osculation with the osculator $E = 2$ is carried out from right to left.

8	1	3	2
5*2+1+8	7*2+1	2*2+3	
19	15	7	

The last number after osculation is 19, so 8132 is divisible by 19.

Since 8132 satisfies divisibility checks for both 2 and 19, it is confirmed to be divisible by 38.

Summary of Divisibility Checks for Divisors Between 2 and 50

Divisor	**Divisibility Check**
2	The number being divided is even.
3	3*3 = 9 or 3*7 = 21 $E = 1$ for positive osculation and $E = 2$ for negative osculation 1 + 2 = 3
4	Divide by 2 two times. The remainder must be 0.
5	The number being divided ends with 0 or 5.
6	Factorize as 3 * 2
7	7*7 = 49 or 7*3 = 21 $E = 5$ for positive osculation or $E = 2$ for negative osculation. 5 + 2 = 7

8	Divide by 2 three times. The remainder must be 0.
9	*E* = 1 for positive osculation.
10	Factorize as 5 * 2
11	*E* = 1 for negative osculation.
12	Factorize as 4 * 3
13	13*3 = 39 or 13*7 = 91 *E* = 4 for positive osculation or *E* = 9 for negative osculation. 4 + 9 = 13
14	Factorize as 7 * 2
15	Factorize as 5 * 3
16	Divide by 2 four times. The remainder must be 0.
17	17*7 = 119 or 17*3 = 51 *E* = 12 for positive osculation or E = 5 for negative osculation. 12 + 5 = 17
18	Factorize as 9 * 2
19	*E* = 2 for positive osculation.
20	Factorize as 5 * 4
21	*E* = 2 for negative osculation.
22	Factorize as 11 * 2
23	23*3 = 69 or 23*7 = 161 *E* = 7 for positive osculation or *E* = 16 for negative osculation. 7 + 16 = 23
24	Factorize as 8 * 3
25	Divide by 5 two times. The remainder must be 0.
26	Factorize as 13 * 2
27	27*7 = 189 or 27*3 = 81 *E* = 19 for positive osculation or *E* = 8 for negative osculation. 19 + 8 = 27
28	Factorize as 7 * 4
29	*E* = 3 for positive osculation.

30	Factorize as 10 * 3
31	*E* = 3 for negative osculation.
32	Divide by 2 five times. The remainder must be 0.
33	Factorize as 11 * 3
34	Factorize as 17 * 2
35	Factorize as 7 * 5
36	Factorize as 9 * 4
37	37*7 = 259 or 37*3 = 111 *E* = 26 for positive osculation or *E* = 11 for negative osculation 26 + 11 = 37
38	Factorize as 19 * 2
39	*E* = 4 for positive osculation.
40	Factorize as 8 * 5
41	*E* = 4 for negative osculation.
42	Factorize as 21 * 2
43	43*3 = 129 or 43*7 = 301. *E* = 13 for positive osculation or *E* = 30 for negative osculation. 13 + 30 = 43
44	Factorize as 11 * 4
45	Factorize as 9 * 5
46	Factorize as 23 * 2
47	47*7 = 329 or 47*3 = 141. *E* = 33 for positive osculation or *E* = 14 for negative osculation. 33 + 14 = 47
48	Factorize as 16 * 3
49	*E* = 5 for positive osculation.
50	Factorize as 25 * 2

Osculation of a Binary Number

Osculation can also be used to check the divisibility of binary numbers. Positive osculators are used for below-base divisors and negative osculators are used for above-base divisors.

Example Check the divisibility of 0110 1100 by 11 (i.e. check the divisibility of 108 by 3).

11 is an above-base number, so the negative osculator $E = 1$ is employed.

$\bar{0}$	1	$\bar{1}$	0	$\bar{1}$	1	$\bar{0}$	0
$0*1+\bar{0}$	$\bar{1}*1+1$	$0*1+\bar{1}$	0*1+0	$1*1+\bar{1}$	0*1+1	$1*0+\bar{0}$	
0	0	$\bar{1}$	0	0	1	0	

The last number after osculation is 0, so the number is divisible by 11.

The table below summarizes divisibility checks for binary divisors between 2 and 10.

Divisor in Decimal	Divisor in Binary	Divisibility Check
2	10	The last digit must be 0.
3	11	Positive osculator $E = 1$ or negative osculator $E = 10$
4	100	The last 2 digits must be 0.
5	101	Negative osculator $E = 10$
6	110	Factorize as 10 * 11
7	111	Positive osculator $E = 100$
8	1000	The last 3 digits must be 0.
9	1001	Negative osculator $E = 100$
10	1010	Factorize as 10 * 101

Conclusion

A divisibility check is a short method to check whether a number is divisible by a particular divisor, without having to actually carry out the division. In mathematics, there is no single algorithm to check the divisibility of potential prime numbers

This paper explains how the Vedic sub-sutra *Veshtanam* ("By Osculation") can be usefully employed to carry out such checks. The factors of a number can then also easily be found. The technique can also be extended to divisibility checks on numbers other than base 10.

References

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