Connection of Paravartya Sutra with Vedic and Non-Vedic Mathematics

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Abstract

Vedic Mathematics is an extremely refined, independent and an efficient mathematical system based on 16 sutras and 13 sub-sutras with simple rules and principles, out of them Paravartya Yojayet is a unique one. The literal meaning of Paravartya Yojayet is “Transpose and apply”. It indicates the interchange of functions and its inverse functions.

The methods discussed, and organization of the content of the paper here are intended to show the importance of the sutra and Vedic Mathematics for modern period and trying to link the application of the sutra with other Vedic Sutras and modern Mathematical methods which are mentioned and not mentioned in the book Vedic Mathematics written by Tirthaji.

Key Words: Paravartya Yojayet, Division, Equations, Solutions, Connections.

1. Introduction

‘Paravartya’ means by *transposing* or *transforming* or *change the setting from one place (or period) to another* or *exchange positions without a change in value*. ‘Yojayet’ means *connect* or *join* or *apply* or *make work for a particular purpose* or *adapt (or conform) oneself to new (or different) conditions*. Therefore, the literal meaning of the formula Paravartya Yojayet is “Transpose and Apply” or “Transpose and Adjust”. Here the word ‘transpose’ indicates the operation of change of the signs like from plus to minus or multiplication to division and conversely. It also indicates the interchange of functions and its inverse functions. Movingfrom one part to another to achieve the solution of the problem is an application of Transpose and Apply or Transpose and Adjust. It is used for both arithmetic as well as algebraic division. It is also used to solve simple linear equations, quadratic equations, cubic equations, partial fractions, etc. In this rule, transposition is related as $+$ to $-$, $\times$ to $\div$, left to right, numerator to denominator etc.and vice-versa

2. Inter-relationships between Paravartya Sutra and other Vedic Sutras

Paravartya Yojayet is Transpose and Adjust or Transpose and Apply. The new form obtained from a given form by interchanging (or, by some operation) is called transpose and to get the result is called adjust (or apply). Vedic Mathematics written by Bharati Krishna Tirthaji is a perfect illustrative as well as explanatory book. So, in this paper we are trying to link the application of the sutra Paravartya Yojayet to modern Mathematics. The following are the examples of how the sutra can be linked to different branches of mathematics mentioned and not mentioned in the book Vedic Mathematics of Tirthaji's.

2.1 Paravartya and Nikhilam

The three Vedic sutras mentioned in Tirthaji's book for division, are: Nikhilam, Paravartya Yojayet and Urdhya Tiryag. Nikhilam and Paravartya sutras are special case division methods, applicable to numbers near the bases, whereas Urdhya Tiryag is general method for division (i.e. for all types of numbers.). Nikhilam method is useful for cases when the divisor-digits are big numbers but is difficult to use when the divisor consists of small digits. To cover these cases, Paravartya Yojayet is useful. The format and working rule of Nikhilum and Paravartya methods are same. We will follow almost the same methods in Paravartya as in the Nikhilum method. However, there is slight difference in process between Nikhilam and Paravartya sutra for division. The table shows the difference of working procedure between Nikhilam and Paravartya Yojayet for division.

Nikhilam Method	**Paravartya Yojayet Method**
We will discuss division by numbers less than the base	We will discuss division by numbers more than the base.
First step will be to find the complement of the divisor. The complement will be a positive number.	First step will be to find the complement of the divisor. The complement will be a negative number.
For example, if the divisor is 98, the complement from nearest base 100 is 2.	For example, if the divisor is 114, the complement from the nearest base100 is -14. This is written as -1-4 by writing each digit of the complement with a changed sign separately.
This sutra works effectively when the first digit of the complement from the base is 1.	This sutra works effectively when the first digit of the divisor is 1.
No readjustment needed	If any digit in the remainder and quotient is negative, the actual result will be readjusted.

2.2 Paravartya and Sunnyamsammya Samuccaye

These are jointly appliedto solve simple linear equations, quadratic equations, cubic equations, and bi-quadratic equations. Here we are trying to show a glimpse of the relation between them. Paravartya formula can tackle the special types of simple equations by merging RHS into LHS under the different headings.

Equations	Solutions by merging with (Using Paravartya)	Using Sunnyamsamaya	Remarks
$\frac{m}{x+a}+\frac{n}{x+b}$ $=\frac{m+n}{x+c}$	$\frac{m(c-a)}{x+a}+\frac{n(c-b)}{x+b}=0$ Then, $x=\frac{mb\,(a-c)+na\,(b-c)}{m\,(c-a)+n\,(c-b)}$	If m (c – a) = n (c – b) then (x + a) + (x + b) = 0	$N_1 + N_2 = N$. But, if $N_1 + N_2 \neq N$, we reform there to make equality by taking LCM
$\frac{m}{x+a}+\frac{n}{x+b}+\frac{p}{x+c}$ $=\frac{m+n+p}{x+d}$	$\frac{m(a-d)}{x+a}+\frac{n(b-d)}{x+b}+\frac{p(c-d)}{x+c}=0$ $\frac{m(a-d)(a-c)}{x+a}+\frac{n(b-d)(b-c)}{x+b}=0$ Then, $x=\frac{mb(a-d)(a-c)+na(b-d)(b-c)}{m(a-d)(a-c)+n(b-d)(b-c)}$	If $m(a-d)(a-c)$ $=n(b-d)(b-c)$ Then, $(x+a)+(x+b)=0$	If $N_1 + N_2 + N_3 = N$
Note: The merger formula can be extended to any finite number of terms.			

2.3 ParavartyaYojayet and Gunita Samuchchaya

These are both jointly applied in the case of special cases of partial fractions, where the denominator of rational fractions has repeated terms but for non-repeated terms we can use only Paravartya sutra.

Types of rational Fraction	***Partial Fraction***	***By using Paravartya/GunitaSamuchchaya***	***Remarks***
$\frac{\emptyset(x)}{\varphi(x)}=\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$	A = $\emptyset(a)$, B = $\emptyset(b)$ *and* C = $\emptyset(c)$	By putting $x = a$, b and c respectively
$\frac{\emptyset(x)}{\varphi(x)}=\frac{px^2+qx+r}{(x-a)^3}$	$\frac{A}{(x-a)^3}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)}$	A=$\emptyset(a)$, B =$\frac{1}{1!}\emptyset'(a)$, $C=\frac{1}{2!}\emptyset''(a)$	By putting $x = a$ in $\emptyset(x), \emptyset'(x), \emptyset''(x)$ Respectively.
$\frac{\emptyset(x)}{\varphi(x)}=\frac{px^3+qx+r}{(x-a)^4}$	$\frac{A}{(x-a)^4}+\frac{B}{(x-a)^3}+\frac{C}{(x-a)^2}+\frac{D}{(x-a)}$	A=$\emptyset(a)$, B =$\frac{1}{1!}\emptyset'(a)$, $C=\frac{1}{2!}\emptyset''(a)$ *and* $D=\frac{1}{3!}\emptyset'''(a)$	By putting $x = a$ in $\emptyset(x)$,$\emptyset'(x)$, $\emptyset''(x)$and $\emptyset'''(x)$ respectively.

$\frac{\emptyset(x)}{\varphi(x)} = \frac{px^3+qx+r}{(a-x)^4}$	$\frac{A}{(x-a)^4} + \frac{B}{(x-a)^3} + \frac{C}{(x-a)^2} + \frac{D}{(x-a)}$	A = $(-1)^4\emptyset(a)$, B =$(-1)^3\frac{1}{1!}\emptyset'(a)$, $C = (-1)^2\frac{1}{2!}\emptyset''(a)$ *and* $D = (-1)^1\frac{1}{3!}\emptyset'''(a)$	By putting x = a in $\emptyset(x)$,$\emptyset'(x)$, $\emptyset''(x)$and $\emptyset'''(x)$ respectively.
Note: **It can be expanded up to finite number of repetitions of the denominator by using Paravartya and Gunita Samuchchaya.**			

Paravartya cn beconnected withother sutras likeAnurupey Sunnyamannyet, Purna Purnabhyam, Lopansthapanabhyam to solves various types of equations.

3. Paravartya Yojayet and Modern Mathematical (conventional) Methods

In his book, Vedic Mathematics, Tirthaji has mentioned a relationship between Paravartya Yojayata and some modern Mathematical methods.

3.1 Paravartya Yojayet and Synthetic Division

The Paravartya Yojayet sutra has a close relation with the Remainder theorem and the Horner process of synthetic division as applied to algebraic division. In the factor theorem, if *f(x)* is divided by $(x – a)$ and $R = f(a) = 0$ then $(x – a)$ is a factor of *f(x)*. Horner's process of synthetic division is similar to the Paravartya sutra; however, it is only a very small part of the Paravartya sutra.

3.2 ParavartyaYojayet and Cramer's Rule

It can be noted that application of Anurupye Sunyamanyat, a sub-sutra (or sub-formula) of Paravartya Yojayet to solve simultaneous linear equations, is similar to Cramer's rule. Consider the simple general simultaneous equations:

$$a_1x + b_1y = c_1;\ a_2x + b_2y = c_2, \text{where } x = \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1};\ y = \frac{c_1a_2-a_2c_1}{a_1b_2-a_2b_1}$$

3.3 Use of Paravartya to solve Simple Equations

Equations	Using Paravartya	Remarks
$ax + b = cx + d$	$x = \frac{d-b}{a-c}$	
$(x + a)(x + b) = (x + c)(x + d)$	$x = \frac{cd - ab}{a+b-c-d}$	$If\ ab = cd\ then\ x = 0$
$\frac{ax+b}{cx+d} = \frac{m}{n}$	$x = \frac{md-nb}{na-mc}$	

$\frac{m}{x+a}+\frac{n}{x+b}+\frac{p}{x+c}=0$	$x=\frac{-mbc-nca-pab}{m(b+c)+n(c+a)+p(a+b)}$ **(if $m+n+p=0$)**	*If $m + n + p \neq 0$, then it will be a quadratic equation and will have to be solved by using different formula.*

4. Paravartya Yojayet and Bhaskaracarya's Mathematics

a) For the equation $ax+b=m$, we transpose a and b such that $x=\frac{m-b}{a}$ (transpose and adjust). This process is written in Lilavati under the topic "Reverse Process".

b) There is also a similarity between a case of BKT's Paravartya Yojayet sutra and division of fraction of Lilavati of Bhaskaracarya (Chapter-13, page-43).

For the division $\frac{p}{q}\div\frac{m}{n}$, it can be changed as $\frac{p}{q}\times\frac{n}{m}$

5. Influence of Paravartya yojayata on modern mathematics

We can find the influence of Paravartya sutra to the modern mathematical results and its conventional methods like the Number system, Functions, Translations, Reflections, Trigonometry, Matrices, Calculus, Vectors and other domains. The following examples show some influence of Paravartya Yojayet within modern mathematics.

(a) We change the verbal problem into mathematical equations and find its solutions, so it is also a transpose and adjust.

(b) The absolute value sign changes to inequality and vice-versa i.e. $|x|\le a \Leftrightarrow -a\le x\le a$ is also taken as transpose and adjust.

(c) The graph of $y=f(-x)$ is the transpose (or reflection) of the graph of $y=f(x)$ about the y-axis. Again, the graph of $y=-f(x)$ is the transpose (or reflection) of the graph of $y=f(x)$ about x-axis.

(d) The function, $f:A\to B$ then $f^{-1}:B\to A$ is called the inverse function.

For example if $y=f(x)=2x-3$ then $x=f^{-1}(y)$ and $f^{-1}(x)=\dfrac{x+3}{2}$.

(e) If $\sin x=k$ (where $k=\sin\theta$ and $-1\le k\le 1$) then $x=n\pi(-1)^n\theta$ for all $n\in\mathbb{Z}$.

(f) $x=\sin y \Leftrightarrow y=\sin^{-1}x$ where $-\frac{\pi}{2}\le y\le\frac{\pi}{2} \Leftrightarrow -1\le x\le 1$.

(g) For matrices A and B, $((A)^T)^T=A$, $(AB)^T=B^TA^T$, and $A^{-1}=\frac{\text{Adjoint of A}}{|A|}$

(i) In calculus, the derivative of a function is a type of transposition.

(j) A definite integral is the transposition of a function into the limit of the sum,

that is, $\int_a^b f(x)\,dx = \lim_{h\to 0} h\{f(a+h)+f(a+2h)+f(a+3h)+\cdots f(a+nh)\}$, and the application is for finnding the area of a plane region under certain conditions..

(k) The vector product of two vectors $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ is a vector normal to the plane

of $\mathbf{a}$ and $\mathbf{b}$ and whose magnitude is equal to the area of the parallelogram with sides equal to the magnitudes of $\mathbf{a}$ and $\mathbf{b}$,

$$\text{that is } \mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

6. Conclusion

Vedic sutra Paravartya Yojayata is an extremely refined, independent and efficient mathematical system The sutra has a remarkable influence in different modern mathematical methods. To study the importance and effectiveness of Vedic method it is better to compare it with conventional methods since mind of the reader or modern mathematical practitioner is already aware of the conventional methods. Little practice of Paravartya Yojayata helps to solve problems comparatively faster and easier. The sutra though has some limitations.

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