

# **The Psychology of Vedic Mathematics – Examples of universal thought patterns**

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## **Abstract**

*In his book, Vedic Mathematics, Bharati Krishna Tirtha claims that the sutras cover every branch of mathematics. The fact that there are sixteen sutras together with a similar number of sub-sutras, raises the question as to what is the nature of these sutras such that they could cover all of mathematics. Many readers of Vedic mathematics may pass over and dismiss this claim merely as an egotistical advertisement for the subject. Here, however, it is taken as a hypothesis worthy of investigation. If his statement is true then it may imply that the human mind, when working in a mathematical context, operates in a relatively limited number of ways or patterns of thinking and that the Vedic mathematical sutras succinctly describe those natural processes. This paper looks at examples of sutras which apply in diverse ways but which unify topics because the mental patterns involved are the same. Mathematics is seen as a human process and is therefore psychological as well as entirely practical. The psychology of mathematics involves recognising patterns of thinking when engaged in mental processes. The sutras also reveal underlying spiritual truths which carry a deeper meaning.*

## **1. Introduction**

This paper addresses questions relating to the nature of the Vedic mathematics of Sri Sankaracarya Bharati Krishna Tirtha. It considers the general nature of the sutras and mathematics as he describes and does not look for references to ancient texts. During the decades since the work was first published in 1965 there have been many attempts to find traces of the sutras in the Parishtas of the Atharvaveda, which was his stated source. Unfortunately, no one has been able to find his source, possibly because not all

Parishtas are published or available to the public, and this has led to a certain amount of criticism of the system<sup>1</sup>. From the outset, however, Tirthaji sets out his interpretation of Veda as having two meanings. One is the collection of ancient texts containing spiritual, as well as practical, guidance. The other is knowledge enlightened by ‘True Realisation’<sup>2</sup>, which describe principles of working. It is through this second meaning that study and development of the subject can progress.

Tirtha's book, *Vedic Mathematics*, is an illustrative volume providing examples of how the sutras work. It never attempts to give a complete and exhaustive description of the extent of any sutra and, considering their multi-faceted characteristics, this is understandable. In several parts of the text he states that topics are to be continued at a later stage and he clearly had the intention of providing further volumes.

Unfortunately, he passed away before being able to provide more material.

There are strong indications that the extent of application of the sutras is unknown and still to be discovered. Not only are there pointers in the text that this is so but further studies<sup>3</sup> have also shown that the sutras have a wider field of application and context. In his introduction Tirthaji gives the most remarkable comment that the sutras apply to all aspects of mathematics and that there is no part of the subject which does not come within their jurisdiction. The question must be asked as to whether, and in what way, this statement could be true? I believe that pursuing an answer to this reveals a new perspective of what mathematics is, of the nature of the human mind and of the harmony between the human psyche and the world around us. The sutras provide us with a new paradigm for mathematics based on practical and experiential knowledge.

## **2. Scope and jurisdiction**

For many years I worked with a group of mathematics teachers researching the Vedic maths system. In addressing the scope and jurisdiction of application of the sutras our studies were guided by two questions. Firstly, what is the sutra operating when performing mathematical activity? Secondly, how does the sutraic treatment of topics described in the text extend and progress? Of course the first requirement in looking for answers was to become thoroughly familiar with the text to see how Tirthaji uses the sutras in addition to analysing what they mean. We quickly discovered that practising Vedic mathematics is a delightful activity. It sharpens the mental faculties, encouraging focus and has a tremendously satisfying effect. We discovered that the key to seeing how the sutras work is to look at what the mind is engaged with at any particular moment of mathematical activity. This gave a direction for pursuing answers to the two questions. It appeared that the sutras described or indicated natural mental processes. As an example, consider adding 198 to 247. A simple and natural method is to add 200 and take away 2, giving the answer 445. This process uses the fact that 198 is deficient from 200 by 2. It is an easy enough method although not taught systematically in schools. Most people will do this naturally. The sutra indicating this is

*Yavadunam* meaning *Deficiency*. Taken in its widest context the sutra describes or indicates the mental process involved with solving any type of problem, mathematical or non-mathematical, which uses a deficiency from a whole.

The work of the group, most of whom were professional mathematicians and teachers, followed a scientific method of accepting the sutras as hypotheses and putting them to the test. The result was a reorientation of the way in which we treated mathematics. It became a *human* activity guided by overarching principles.

### **3. Patterns of cognition**

One of the outstanding features of Vedic Mathematics is that it provides a system for patterns of cognition in both the experience of processing and in the structure of formulae. The aphorisms have a wide range of modes of application. They provide general rules for problem solving, specific instructions for calculation and algebraic manipulation, rules of thumb and patterns of working. Some of the sutras appear to have very specific applications whilst others express overarching principles. They can also carry more than one of these modes simultaneously.

The relatively small number of sutras suggests that the human mind has a similar limited range of channels through which it operates. Just as in the musical octave, where a small number of musical and harmonious relationships lead to a

vast, perhaps infinite, variety of musical compositions, so do the sutras of Vedic maths have a huge scope and potential.

### **4. Giving the same name to different things**

Much of mathematics concerns finding patterns. In his book *Prelude to Mathematics*, W.W.Sawyer states, "...in nature we sometimes find the same pattern again and again in different contexts, as if the supply of suitable patterns were extremely limited"<sup>4</sup>.

A similar view was expressed by Poincare when he said, "Mathematics is the art of giving the same name to different things"<sup>5</sup>.

To investigate mathematics in this way is a move towards unity, away from diversity. The sutras of Vedic mathematics powerfully demonstrate the same principle. A single sutra can describe a pattern of mental process, or shape of a formula, which is then repeated in disconnected topics. The topics then become unified. The lack of logical connection in terms of hierarchical structure indicates that when dealing with the aphorisms a certain mental looseness, flexibility and open-mindedness is necessary. Sri Tirtha emphasises this point when he says, "we are called upon to enter on such a scientific quest as this, by divesting our minds of all pre-conceived notions, keeping our minds ever open, in all humility (as humility alone behoves and befits the real seeker after truth), welcoming the light

of knowledge from whatever direction it may be forthcoming.”<sup>6</sup>

This is not a rigid analytical approach, which is largely modeled after the logic exemplified in Euclid’s Elements. In Euclid any proposition is validated by logically stepping from previously determined propositions and eventually back to the postulates, definitions and axioms which he sets out at the beginning. Inasmuch as we understand axiomatic as meaning that a statement is a self-evident principle some of the Vedic mathematical sutras are treated in the same way. The key to understanding this lies in appreciating that the sutras are not abstract statements of logic but are principles experienced at the personal level.

### **5. Paravartya Yojayet**

The Paravartya sutra may provide some useful insights. It is the most frequently quoted sutra in *Vedic Mathematics* which Tirthaji translates as *Transpose and Apply* and *Transpose and Adjust*. It has numerous applications, such as, in finding equations of straight lines, algebraic division, division of fractions, dividing a line into equal parts, solving equations, equations of lines and transformations. The immense diversity of its scope indicates a flexible formula which is a cognitive pattern.

### **6. Patterns of formulae**

Not only does the Vedic system display patterns within processes but it also provides patterns of

form. This appears in the shape and structure of mathematical formulae. It is not uncommon to find formulae with the same basic shape but which apply to and deal with entirely different contexts. A simple example is the ‘rule of three’ in which three variables are related in elementary physics and mathematics. There are a number of formulae which express a simple relationship of proportion and, in fact *Anurupyena - Proportionately* is the sutra under which they all come. Such examples are speed = distance/time, density = mass/volume and pressure = force/area. The identical shape of these formulae render them easy to learn. Even though the specific details of each formula may not be related, the formulae are connected because they have the same form or shape.

### **7. Gunita Samuccaya**

This congruence of form or shape is an intrinsic feature of Vedic mathematics. This can be seen with the case of the *Gunita* sutra which means Product/Sum. The full wording of the sutra in Sanskrit is *Gunita Samuccaya Samuccaya Gunita* and the literal translation is *Product Sum, Sum Product*. The word *Gunita* usually means product or some form of connecting, augmentation or filling<sup>7</sup> [MW], and a *Samuccaya* is a sum, heap or collection. This sutra is usually taken as *The product of the sum equals the sum of the product* but it is an overarching descriptor for a number

of processes or patterns of working as well as for various mathematical formulae.

A simple numerical application applies to the product of two numbers wherein the product of the digital roots is equal to the digital roots of the product.

The digital root of a number is the summation of its digits continued until there is one digit remaining. For example the digital root of 35 is  $3 + 5 = 8$ . The digital root of 769 is 4,  $7 + 6 + 9 = 22$  and  $2 + 2 = 4$ . The digital root is also the remainder when the number is divided by 9.  $769 \div 9$  is 85 remainder 4.

With the product of two numbers, such as  $35 \times 21 = 735$ , the digital roots of the two numbers are 8 and 3. The product of 8 and 3, 24, has digital root 6. The digital root of 735 is also 6. This formula is also applicable to all cases of addition, subtraction and division. Perhaps a more useful application is for checking factors and products within algebraic identities. Here the wording the formula is slightly different and this is the example Tirthaji gives in his book.

*The product of the sum of the coefficients in the factors equals the sum of the coefficients in the product.*

$$(2x + 3)(5x - 7) = 10x^2 + x - 21$$

The product of the sum of the coefficients in the factors is  $5 \times -2 = -10$ . The sum of the coefficients in the product is  $10 + 1 - 21 = -10$ .

Below is a list of some of the applications of the Gunita sutra all with a common form:

1. In complex numbers,

$$\text{Arg}z + \text{arg} w = \text{arg}(z \times w)$$

$$\text{and } \text{mod}(zw) = \text{mod}(z) \times \text{mod}(w)$$

2. In matrices  $\det(A \times B) = \det A \times \det B$

3. In logarithms,  $\log(AB) = \log A + \log B$

4. De Moivre's theorem,

$$(\cos x + i \sin x)^n = \cos nx + i \sin x$$

5. In set theory,  $(A \cup B)' = A' \cap B'$  and vice versa

6. In calculus,

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

$$\text{and } \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

7. Centre of mass,  $\bar{x} \sum_{i=1}^n m_i = \sum_{i=1}^n m_i x_i$

In each case the specific wording in English may be slightly different. For example in number 7, *The sum of the moments equals the moment of the sum.* In number 2, *The determinant of the product equals the product of the determinants.* In these examples there is a similarity which may be described in a general way as, *The process on the aggregate is equal to the aggregate of the process.* Diverse topics then become connected through similarity of form.

## 8. Intuition in mathematics

Intuition and logic both have their part to play in mathematics. Here is a description by the famous French mathematician Henri Poincare:

*It is impossible to study the works of the great mathematicians, or even those of the lesser, without noticing and distinguishing two opposite tendencies, or rather two entirely different kinds of minds. The one sort are above all preoccupied with logic; to read their works, one is tempted to believe they have advanced only step by step, after the manner of a Vauban who pushes on his trenches against the place besieged, leaving nothing to chance. The other sort are guided by intuition and at the first stroke make quick but sometimes precarious conquests, like bold cavalrymen of the advance guard.*<sup>8</sup>

Mathematicians frequently use intuition to solve problems. It is an invaluable tool. There is a sutra to cover this scenario. It simply states

*Vilokanainaiva*, which is Sanskrit for *By means of simple observation*, or *By inspection*. It deals with the case in which you solve a problem simply by looking. For example, for many people a simple equation like  $x + 3 = 40$ , can be solved simply by looking and visualizing the answer.

The experience is in seeing the answer straight away and of course it is always then possible to justify the answer through logical steps. But this process is not limited to mathematics. There are many everyday scenarios in which we are

presented with a problem and the solution is found just by observation. Using observation to solve problems is a common experience but, of course, is an entirely personal affair.

Here is an anecdotal example. An old friend of mine is a retired water engineer. He was very successful at his work and was frequently employed by Thames Water to handle very large projects, such as the sewer system in Cairo and the pollution problem of factory waste products along the lower Ganges in India. He once told me the way he solved problems in engineering was to sit by the water, fall still within himself, and just watch. He found the solutions to many water engineering problems working in this way - just through stillness and observation. I saw this at first hand once when we were doing some charity work together. One task was to thread an electric cable through a rigid 100 metre plastic pipe which was about 60mm in diameter and lying horizontally on the ground. I thought I was good at solving practical problems but could not figure this one out. He, on the other hand, just observed and waited for the answer. Sure enough, after a short while the solution came to him. His idea was to half fill the pipe with running water and have a small float inside which would pull a long piece of string through. The string would then be used to pull the cable. He made the float and tested its buoyancy. He then blocked one end of the pipe except for two holes; one to let in the

running water from a hose and the other much smaller to let in the string. The float was inside and attached to the string. When he turned the hose on he said, "let's walk". He seemed to know how fast the float would travel and, sure enough, when we got to the other end we saw the float emerge.

What one person can solve merely by looking is not necessarily the same as what another person can do. So it is a common experience yet played out in immense variety. Some might argue that this is not mathematics at all but as Poincare points out mathematics must include intuitional experience at a personal level.

Here is another problem which demonstrates the use of observation or inspection. Suppose the Wimbledon Lawn Tennis knockout competition for men's singles has 128 contestants. How many matches are there in that competition altogether? Two of the routes to the answer are very different. The first route is to simply visualize that there must be 127 matches. So what is the reasoning behind the experience of the instant answer? It is this. In a knockout competition there is one winner and 127 losers. Every match has one loser and so there are 127 matches. The second route is to see that in the first round there are 64 matches, in the second round there are 32 matches, then 16, 8, 4, 2 in the semi-final and 1 in the final. Then add up these numbers,  $64 + 32 + 16 + 8 + 4 + 2 + 1$ , and come to the answer 127. The fluidity of

the Vedic maths sutras provides us with different ways of dealing with problems. In the case above, some people might solve it just by inspection, whilst others will use the analytic process. And you can combine processes by saying "The answer just came to me but here is the reasoning!" Mathematics frequently has both a rational and intuitive connectedness. For example, when you see the digits of the square root of 2 in pairs, as 14, 14, 21, 35... you may intuitively think that this has something to do with the seven times table. You would be right! But what that connection is exactly and why the next pair of digits is not 63 but 62 is found through applying rational processes. A natural response to seeing patterns is to go with the flow, so to speak, and then at a later stage see if your hunch is correct and supported by logical argument. Going with the flow is an intuitive process and is as much part of mathematical thinking as the use of logic.

### **9. Mental processes not limited to mathematics**

As to whether or not a sutra is applicable only to mathematics or has a wider use is not an issue once we accept the flexible nature of their applications. It merely indicates that the sutras tell us something about how the mind naturally responds in various situations. An example of this can be seen in how an architect deals with designing a loft conversion. An experienced architect may have a mental blueprint or template

which forms the basis of the design. This is because very many loft conversions are constructed within houses of similar design. So the architect has the template and, looking at the particular task in hand, makes adjustments to the plan. This process is encapsulated by the Paravartya sutra mentioned earlier.

### **10. Elements of spirituality**

Some of the sutras appear to have a deeper meaning than just describing a mathematical formula or process. *Paravartya Yojayet*, for example, has manifold applications in relation to transpositions and transformations but the literal translation is, *Turn back and unite*. This is one way of understanding a central tenant of Advaita philosophy. In terms of spiritual evolution *Turn back and unite* can mean the relinquishing of desires and attachments so as to move closer to realising the unity of one's Self. The same maxim is indicated in the Christian teaching through the parable of the prodigal son. The son takes his inheritance and wastes it on riotous living. Eventually coming to his senses he goes back to his father to be united once again.

Another example is *Vyashti Samashti*. In Vedic mathematics this has to do with finding the average of a set of numbers. Interestingly, the mean value of a set of numbers is a single number which, in some way, represents the whole. It is an individual through which the whole is reflected. The sutra also describes the

essential nature of self-similarity found in nature and throughout chaos theory. Literally, the Vyashti is an *individual* or *the Being as one* and Samashti is *the universal Being*. It comes from the verb *As*, to be. The prefix *Vi* carries the sense of *differentiation* and the prefix *sam* means *altogether as one*. The sutra has the sense of the universal reflected in the individual and is akin to the ancient teaching of *As above, so below*, found in Platonic philosophy, the Abrahamic religions of Judaism, Islam and Christianity, the Hermetic tradition and in ancient Egypt.

When learning Vedic mathematics for the first time most students come across multiplication by *All from nine and the last from ten*. The sutra is *Nikhilam Navatascaraman Dasatah* and the translation is accurate. Using the sutra reveals how easy some problems become when relating numbers to a power of ten. Any number, such as 10, 100, 1000, and so on, is a unity. In using the sutra the student relates each number to that unity. In a wider context there is much merit in this in order to solve problems. Most of the worlds problems seem to arise from divisive forces indicating a lack of unity. The reverse of this is described in the Isha Upanisad, "Of a certainty the man who can see all creatures in himself, himself in all creatures, knows no sorrow."<sup>9</sup>

### **11. Conclusion**

Looking at the mental processes taking place whilst performing mathematics and also at



similarities within the shapes or structure of formulae requires taking a step back away from the involvement with results and functionality. Questioning the nature of the Vedic maths sutras leads us in this direction. It requires us to distinguish between the shapes of nature and the seeds of nature. It leads to an appreciation of an underlying unity which connects the conscious processes of mathematics, the structure of mathematics and the world around us to which it is applied and through which we appreciate its reality. And the harmony appearing in the world around us is not entirely separate from human intellect.

Henri Poincare refers to this in the *Value of Science*, “Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt, a reality completely independent of the mind which conceives it, sees or feels it, is an impossibility.”<sup>10</sup>

The sutras of Vedic mathematics are like seeds from which manifold applications develop. Practice and reflection on the sutras and an appreciation of the underlying cognitive patterns yield a deeper penetration into their meaning. It is through this that we may come to understand more fully Sri Tirtha’s overarching statement concerning their universal jurisdiction.

## Bibliography

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