Vyashti Samashti – A sutra from Shankaracarya Bharati Krishna Tirtha

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Abstract

This mathematical sutra from the Vedic Mathematics of Shankaracarya Sri Bharati Krishna Tirtha has manifold applications in algebra, induction, statistics, dynamical geometry and chaos theory. It describes how the form of the whole is reflected in the specific or individual. It also has much wider significance as it reflects a common philosophical principal in many ancient and modern traditions. This principle is found in teachings from Vedanta, ancient Egypt, Christianity, Hermeticism, Judaism, and Platonism. One of the important characteristics of Vedantic philosophy is that the practical and spiritual are not separate but are interrelated and holistic. This sutra provides a prime example of how practical knowledge is related to a spiritual principle.

This paper discusses some of the mathematical applications of the sutra and aspects of the philosophical meaning in terms of how the Samashti, or Universal, is reflected within the Vyashti, Individual.

Introduction

Vyashti Samashti is a wide-ranging sutra with many applications but in essence it postulates a harmonious relationship between a part and a whole. It is only mentioned once in Swami Tirtha's book[1] in connection with solving a very particular type of equation in which the average of two terms is used to provide an easy solution. Given that his text is illustrative and that each sutra has manifold applications it is worth considering other avenues where the sutra applies, what it means at the level of Mathematics and if there are any other significant applications. The aim of this paper is to show that such a sutra not only has multifarious applications in Mathematics but also meanings related to the major philosophical and religious traditions. It reveals that mathematics is not an isolated field of abstract thinking but is more an integrated aspect of human psychology.

The concept expressed in this sutra is simple, subtle and yet wide-ranging. The meaning is *Specific and General*, or *Individual and Total*. Considering the calculated mean of a set of numbers, it is a single number that, in some way, represents the whole. For example, to calculate a simple mean of 2, 4, 6, 8, 10, the total of all the numbers in the set, that is 30, is divided by the number of individual numbers in the set, 5, giving 6. The mean, 6, reflects the whole in some way. But it is not the only average. In statistics other averages are used such as median and mode. The sutra also applies to these because it is an overarching aphorism. In a broader sense in Mathematics the sutra operates whenever there is a situation in which the whole is reflected in the part or an individual.

Before looking at some of the other applications it is worth considering what the words of the sutra mean. Vyashti, in Sanskrit, simply means individual and is constructed with the prefix Vi, meaning *differentiated*, and ashti, meaning Existence[2]. So it literally means, the *Existence differentiated*. And this meaning follows the philosophical tradition of Vedanta in which each individual being is a reflection or a part of the universal being. Samashti is made from the

prefix Sam, meaning *altogether as one*, and ashti – e*xistence[3]*. So it comes to mean *Universal Existence*. Derivative meanings are Totality, Cosmos, Brahma, and Universal.

The general meaning of the sutra is found wherever the whole is reflected in the part or individual in some way and there are many instances not only in mathematics but also in everyday life where this happens. It is often called the principle of self-similarity.

Mathematical Applications

Mischievously, this sutra only has one mention in the text of Swami Tirtha's book and that is in relation to a certain special type of biquadratic equation described in Chapter 19.

An example is to find the solution of the biquadratic, $(x+7)^4 + (x+5)^4 = 706$ in which two binomials on the left are both raised to the 4th power and there stands a single number on the right. The average of the two binomials is used to break the equation down into a simpler one.

$$(x+7)^4 + (x+5)^4 = 706$$
$$\text{Let } x+6 = a$$
$$(a+1)^4 + (a-1)^4 = 706$$
$$2a^4 + 12a^2 + 2 = 706$$

$$\text{From which, } a = \pm 4 \text{ or } \pm\sqrt{22}$$

There is a pleasing application of this sutra within calculus concerning the derivatives of combined functions, such as,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x \qquad \text{and} \qquad \frac{d}{dx}(3x^4 - 3x + 7)^5 = 5(12x^3 - 3)(3x^4 - 3x + 7)^4$$

Conventionally this is taught and practised using the chain rule, which is an application of the *Transpose and Adjust* sutra. When students have understood and practised the chain rule they usually repeatedly use the pedantic method as if proving it every time they want to use it. But the sutra can be used to obtain the answer in one step.

In this context the sutra can be read as, *the differential of the whole times the differential of the part* or, more usefully, *the differential of the outside times the differential of the inside*.

More general forms are:

$$\frac{d}{dx}\left(f(x)^n\right) = f'(x) \times nf(x)^{n-1}$$

$$\frac{d}{dx}\left(\sin f(x)\right) = f'(x) \times \cos f(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) \times e^{f(x)}$$

$$\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}$$

Proof by induction constitutes a further application of the sutra in combination with *By one more than the one before*. Here is an example:

Prove that the sum of the first *n* integers is given by $\sum_{1}^{n} n = \dfrac{n(n+1)}{2}$.

For $n = 1$, $\dfrac{n(n+1)}{2} = \dfrac{1 \times 2}{2} = 1$. Therefore it is true for $n = 1$.

Assume it is true for $n = k$. Then $\sum_{1}^{k} n = \dfrac{k(k+1)}{2} = \dfrac{k^2 + k}{2}$.

Adding the next term,

$$\sum_{1}^{k} n + k + 1 = \dfrac{k^2 + k}{2} + k + 1 = \dfrac{k^2 + 3k + 2}{2} = \dfrac{(k+1)((k+1)+1)}{2}$$

And so it is true for $k + 1$. Since it is true for $n = 1$ and, if true for $n = k$, then also true for $n = k + 1$, then it is true for all *n*.

The proof is based on showing that the proposition is true for a particular value of *n*, assuming it is true for a general value of *n*, called *k*, and then showing that it will then also be true for the succeeding value of *n*. This then shows that it is true for all values of *n*.

Another type of application occurs when a single entity gets referred to the whole, in order to solve a problem. Here is an example. Suppose Wimbledon Men's Singles tennis competition has 128 players. It is a knockout competition. How many matches are there in the whole competition from the first round through to the final? One method is to figure that there are 64 matches in the first round, 32 in the second round and so on until one match in the final. The sum of each round will give the answer. But the sutra can be employed to provide a more intuitive answer. The argument goes like this: each match has a loser. The winner of the competition is the only one who has not lost and so there are 127 losers and hence 127 matches.

The concept within Vyashti Samashti lies at the heart of the whole realm of probability and statistics. Much of statistical theory is based on the issue of providing a simple representation of a large amount of data. That representation can take the form of a mean, a regression line, a range, a distribution, and so on. It never gives a complete picture but shows some aspect of the whole.

The normal distribution curve is intriguing because it seems to connect each individual to the whole.
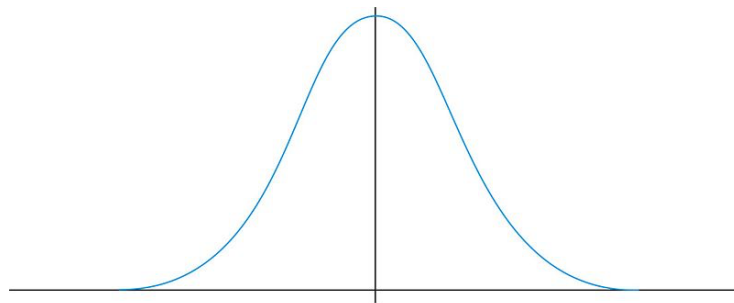
Figure 1. The normal distribution curve

Take, for instance, the unbiased weight of fish of a particular species caught by fishermen in a trawler. The weights will naturally be distributed in accordance with normal distribution.

The standardised normal distribution has equation,

$$y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

Any particular fish will have its weight as part of the distribution and so contributes to the whole. This begs the question as to the very nature of individuality. Is the fish really an individual?

The concept of the normal distribution has become so universal that it now provides the basis of almost all parametric statistical methods. And yet its basis is theoretical. It is a mathematical reality that appears to have a strong correlation with empirical distributions.

Another appearance of the sutra concerns the concept of the centre of mass of a body. This is regarded as a single point forming the average of all individual particles that make up the body. In terms of weight, the centre of gravity is the single point through which the entire weight of a body can be seen to act.

The following equation expresses this, as the moment of the sum of masses is equal to the sum of the moments of the masses.

$$\bar{x} \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} m_i x_i$$

Self-Similarity

Vyashti Samashti encapsulates the principle of self-similarity. This lies at the core of spatial harmony and is evident in much of traditional arts, particularly as an ever-present characteristic of Islamic geometry.
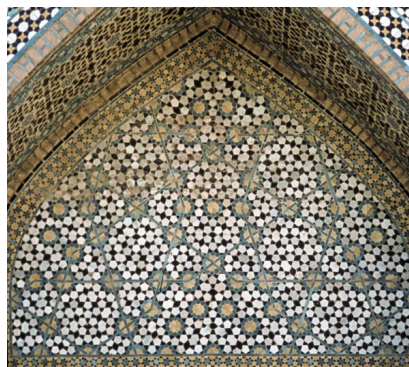
**Figure 2. Portal from the Darb-I Imam Shrine at Isfahan, Iran (1453 CE)**

So inherent is this principle to the way we think and deal with practical matters that it can often go unnoticed. Consider how a retailer determines percentage profit for a single item based on the costs involved for the whole company or how a decision is made for an individual based on the needs of the whole society. Interior designers will frequently relate a single item within a room to the design of the whole room. An example from architecture is

Robie House in Chicago designed by Frank Lloyd Wright in 1908[4]. Wright believed in designing structures that were in harmony with humanity and its environment, a philosophy he called organic architecture. The basic principle of organic architecture is not that *form follows function* but that *form and function are one*[5]. Robie House is the most famous example of the Prarie style in which the flat planes of the Prarie are used to inspire the design of open spaces.. This house is also said to exemplify the use of self-similarity in architecture.
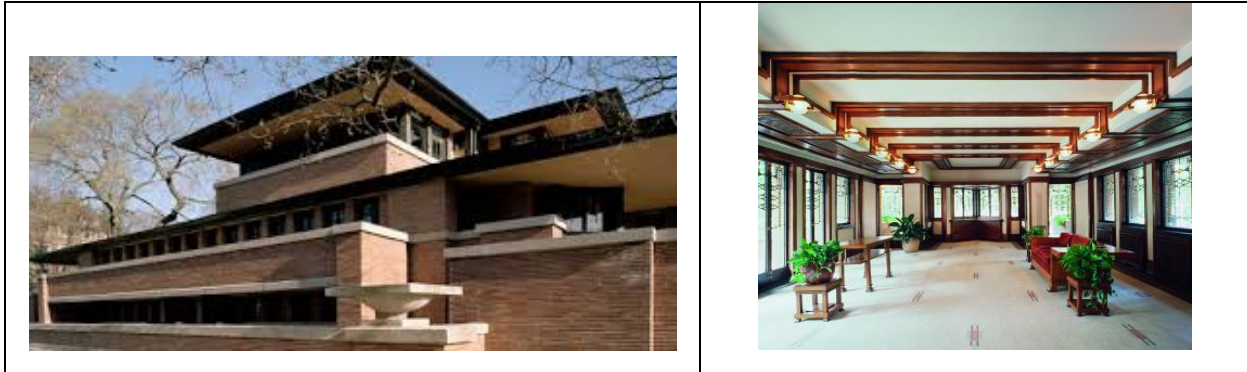
Figure 3. Robie House, Chicago

Self-similarity is also found in the natural growth law expressed by the golden mean which forms a fundamental backbone for spatial and numerical relationships in the biosphere. A simple geometric example can be seen with the golden rectangle.
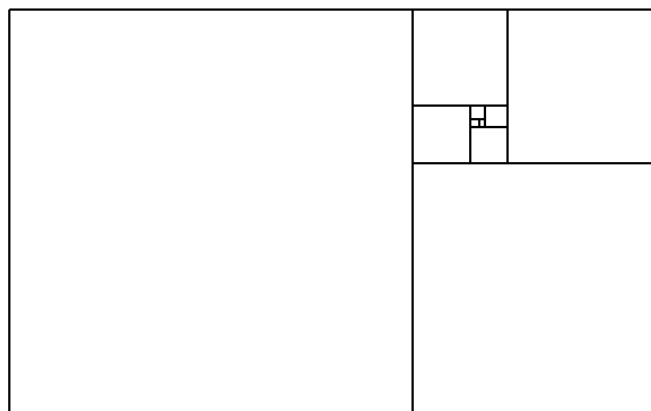

Figure 4. The golden rectangle

This rectangle is such that in cutting off a square the remaining rectangle has the same unique proportions as the original. It is also an example of another sutra, Śishyate Śeśasamjñah – *the remainder remains constant*.

Self-similarity is also a key feature of the mathematics described by Chaos theory and fractals.

Examples of fractals can be seen in crystals, in which the same and only building block (called an elementary cell) is repeated over and over again, tiling the entire space[6].

Figure 3. Ice-ferns
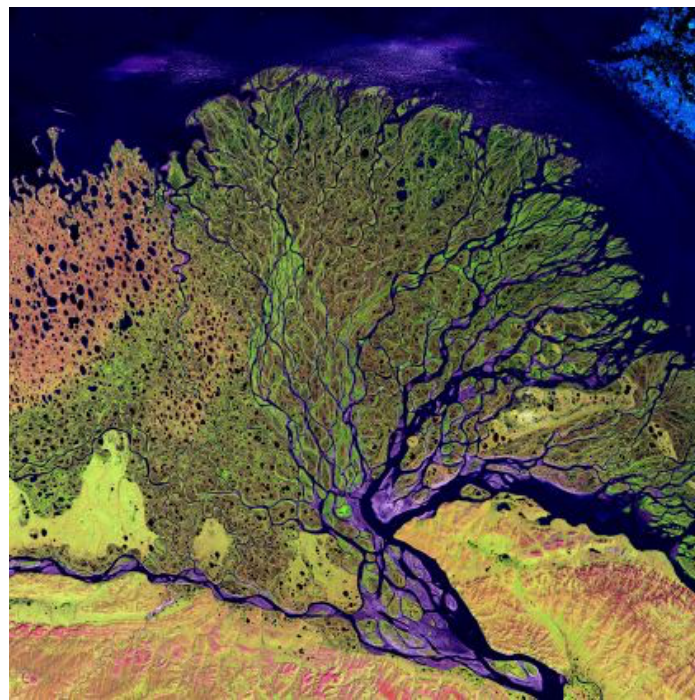
Figure 4. Lena River delta, Siberia

Other examples of geometric self-similarity exist as a conspicuous feature of many plants. Mathematically these are expressed in terms of affine transformations that map a structure into its components[7].

A fractal is a natural phenomenon or a mathematical set that exhibits a repeating pattern that displays at every scale[8]. The mathematical development of fractals began with Leibnitz in the 17th Century but it was in the latter half of the 20th century that they received more formal treatment and now form part of Chaos Theory.

In the Asymmetric Cantor Set, see Figure 4, the initial line is divided into three parts, half of the line on the right and two quarters on the left with the second quarter subtracted. The two remaining black lines each have the process repeated.
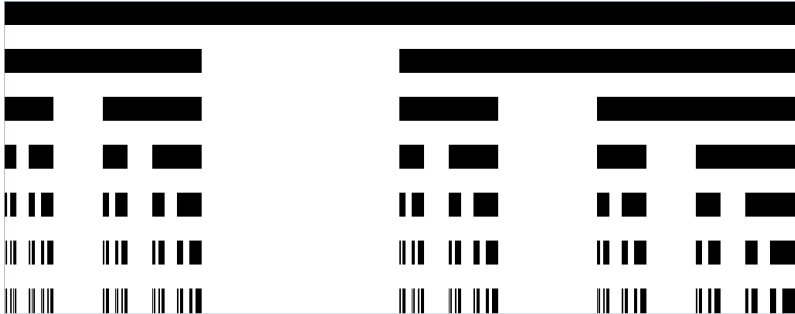

Figure 4. Cantor Set

Another example of a simple fractal is the pentaflake in which a pentagon is divided into six pentagons. This is repeated again and again producing smaller and smaller pentagons.
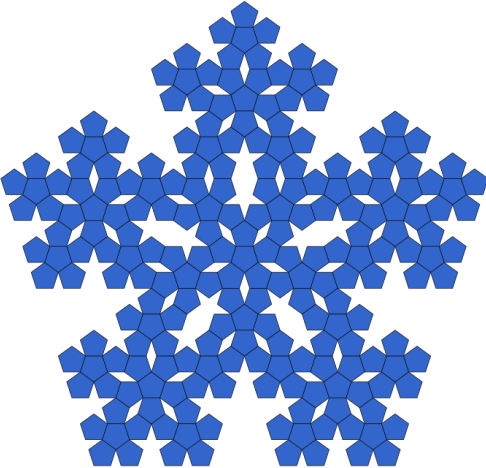

Figure 5. Pentaflake

Psychology

An inherent feature of human psychology is the ability to cognise entities as being whole and single or as being plural. In other words we can view something as *being one* or as *being many*. For example, in looking at a tree we may see it as consisting of a huge number of leaves and twigs, and so forth, or we may see it as a single tree. The same is true with regard to number itself. When we think of the number five, for instance, we may view it as a single entity but also as made up of five ones or any other number of parts. This reflects the integral relationship between the whole and the parts and gives further meaning to the sutra.

Philosophy

The principle of Vyashti Samashti has a deeper significance than just mathematical as it reflects a common philosophical and religious principal that features within many ancient and modern traditions. It is found in teachings from Vedanta, Christianity, ancient Egypt, Hermeticism, Judaism, and Platonism.

In relation to Vyashti Samashti the Vedantic teachings are most accessible because one of the important characteristics is that the practical and spiritual are not separate but are interrelated and holistic. This sutra provides a prime example of how practical knowledge is related to a spiritual principle. In Vedanta, Vyashti Samashti signifies an identity between the microcosm, consciousness as the foundation of the individual, and the macrocosm, consciousness as the foundation of the universal. This identity is described in Vivekchudamani of Adi Shankara and forms the cardinal tenet of his text, "Brahman alone is real, the universe is unreal and the individual soul (Vyashti) is no other than the Universal Soul (Samshti)."[9]

Commentaries on the Vedas provide various analogies used to assist with understanding what is meant. The sun shines and its light is reflected in many pots containing water. This is called the Pratibimba (reflection) analogy. The observer sees many images of the sun but, in reality, there is only one. The individual pots are made of clay and, whilst they are formed they are seen as individual pots. The pots have relative existence but have a single substance as their substratum. A wave on the ocean has its individual characteristics but is still part of the ocean and can never be separate from it.

Similar to the analogy of the sun shining in pots of water as images is found in the Abrahamic teaching, that is, Judaism, Christianity and Islam. In the book of Genesis we read, "So God created Man in his own image, in the image of God created he him; male and female created he them." Many Jews, Christians and Muslims interpret this as a statement not just of origin, but as the philosophy of "as above, so below", that the spirit and nature of the One is reflected in each and every individual.

And this idea of image is also described in Book 6 of Plato's Republic in which he depicts the world as being divided into the Intelligible and the Visible. For example, consider a circle. It is a form of knowledge and our concept of circle can be completely perfect. The circles we see in the visible world are never perfect. In Plato's terms they are like copies or reflections of pure knowledge and if it was not for the existence of the concept of "pure" circle we would never recognise them when seen.

In Hermeticism, the ultimate reality is referred to variously as God, the All, or the One. God in the Hermetica is unitary and transcendent; he is one and exists apart from the material cosmos.[10] The Emerald of Hermes Trismegistus explicitly refers to the principle, "That which is Below corresponds to that which is Above, and that which is Above corresponds to that which is Below".[11] Similarly, "As above, so below" is a primary principle of Kabbala, the esoteric school of thought originating in Judaism.[12] It also appears in Christian teaching when Jesus Christ says "I am my father are one" (John 10:30).

Conclusion

It is easy to see how the Vyashti Samashti sutra conveys a meaning far wider and deeper than as used in mathematics. It is relevant as a spiritual principle and as a practical principle that can be used to solve problems. Swami Tirtha gives some indication of the integration of spiritual and practical knowledge. He says, "…the Vedas… contain within themselves all the knowledge needed by mankind relating not only to the "so called" spiritual matters but also to

those usually described as purely "secular". Of secular subjects he points out that "[they] are inherent parts of the Vedas, ie., they are reckoned as "spiritual" studies and catered for as such therein."[13]

The Vyashti Samashti sutra provides us with a real glimpse how we may understand the unity of the practical and spiritual.

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