## IAVM

## IVMO 2021 Intermediate <br> Time allowed-1 Hour

1. What are the Highest Common Factor and Lowest Common Multiple of 64 and 72 ?
A 4 and 1152
B 8 and 576
C 36 and 144
D 8 and 512
E 16 and 288
2. By continual summing the digits of a number, you arrive at the digital root. What is the digital root of 23648157185 ?
A 1
B 3
C 5
D 7
E 9
3. The number, $4 \overline{2} \overline{4} 30 \overline{7} \overline{3}$, has four vinculum digits. What is the correct number when the digits are expressed as positive integers?
A 3752927
B 3862937
C 4862037
D 4753126
E 3762927
4. In using Nikhilam multiplication, which of the following is correct?
752-258
752-357
752-248
752-246
$752+248$
A $\frac{\times 997-003}{749744}$
B $\frac{\times 997-002}{749744}$
C $\begin{array}{r}\times 997-003 \\ 749744\end{array}$
D $\frac{\times 997-004}{749744}$
E $\frac{\times 997+003}{749 \quad 744}$
5. What are the final three digits of $9999996^{2}$ ?
A 992
B 016
C 486
D 996
E 136
6. How many zeros come after the decimal point in the calculation, $0.00000075^{2}$ ?
A 6
B 8
C 10
D 12
E 14
7. What are the vinculum digits in the calculation, $1023 \times 991$ ?
A $0 \overline{2} \overline{3}$
B $00 \overline{8}$
C $\overline{2} 0 \overline{7}$
D $\overline{2} \overline{1} \overline{9}$
E $\overline{7} 0 \overline{3}$
8. When using Vertically and crosswise to calculate $2361 \times 4215$, what is the result of the fourth step before any carry digits are included?
A 29
B 16
C 31
D 32
E 23
9. $12121212121 \times 11$
A 1333333331
B 13333333331
C 133333333331
D 1333333333331
E 13333333333331
10. Which fraction is the smallest?
A $\frac{1}{2000000008}$
B $\frac{2}{4000000015}$
C $\frac{4}{8000000036}$
D $\frac{8}{16000000056}$
E $\frac{2}{4000000017}$
11. When using a working base of 250 and a real base of 1000 to work out $251 \times 257$, what operation is needed for the left-hand part of the calculation?
A $\times 2.5$
B $\div 2.5$
C $\times 4$
D $\div 4$
E $\times 8$
12. The product of two prime numbers is 2021 , which can be worked out using the rule, When the final digits add to 10 . What are the two priime numbers?
A $79 \times 61$
B $83 \times 17$
C $37 \times 23$
D $53 \times 67$
E $43 \times 47$
13. A bottling factory produces 19,866 bottles in 42 hours. In using straight division to find how many bottles are produced each hour, what is the second to last step?
A $(38-26) \div 4$
B $(26-14) \div 4$
C $(32-20) \div 4$
D (14-2) $\div 4$
E $(86-74) \div 4$
14. What is the integer remainder when 1245003411 is divided by 999998 ?
A 9510
B 5901
C 1059
D 5190
E 1095
15. $1000004 \times 1214623$
A 1214627858492 B 1214628858492 C 1214627868492
D 1214627859492 E 1214627858482
16. What are the last four digits in the recurring pattern in the decimal equivalent of $\frac{1}{19}$ ?
A 6667
B 0526
C 8421
D 9991
E 3829
17. In calculating $31^{3}$, which partial sum is correct?
A

| 13927 |
| :--- | :--- | :--- |
| 618 |

B
27931
279
C

| 27931 |
| :---: |
| 927 |

D 27931 186
E None of these
18. What is the sum of the first twenty odd numbers?
A 200
B 300
C 400
D 500
E 600
19. How many lines of symmetry does a regular dodecagon ( 12 sides) have?
A 8
B 12
C 16
D 24
E 36
20. What is the total area of the shaded regions?

A 28
B 35
C 70
D 42
E Not enough information
21. Lisa Doolittle converted the partly recurring decimal into a fraction. In which step did she make a mistake?

$$
\begin{array}{rlrl}
\text { Let } x & =0.2 \dot{4} \dot{5} \\
\text { A } & 1000 x & =245 . \dot{4} \dot{5} \\
\mathrm{~B} & 100 x & =2 . \dot{4} \dot{5} \\
\mathrm{C} & & 900 x & =243 \\
\mathrm{D} & x & =\frac{243}{900} \\
\mathrm{E} & & x & =\frac{81}{100}
\end{array}
$$

22. There are 100 students in a class. Exactly $99 \%$ of them have their books. How many students with books must leave the room to bring down the percentage with books to $98 \%$ ?
A 1
B 2
C 10
D 49
E 50
23. A waiter's pay consists of tips and a salary. During one month, his tips were $5 / 4$ of his salary. What fraction of his pay came from tips?
A $\frac{4}{9}$
B $\frac{5}{9}$
C $\frac{4}{5}$
D $\frac{9}{5}$
E $\frac{9}{4}$
24. Harry the hedgehog was timed walking from one end of the garden to the other. He covered 11 metres in 19.6 seconds. Approximately, how fast is this in km per hour?
A 0.5
B 1
C 2
D 4
E 8
25. The perimeter of a right-angled triangle is 72 cm . The lengths of the sides are in the ratio $3: 4: 5$. What is the area of the triangle in $\mathrm{cm}^{2}$ ?
A 144
B 216
C 270
D 360
E 1620
26. A box contains red, blue and yellow counters. The ratio of red to blue is $3: 5$ and the ratio of blue to yellow is $7: 9$. There are 90 yellow counters in the box. How many red counters are there?
A 21
B 35
C 42
D 45
E 63
27. The cross in the diagram has one vertical line of symmetry. All measurements are in centimetres. What is the perimeter?
A 24.4
B 45.2
C 52
D 60
E 76

28. Which of the following is a possible solution to the equation, $x+\frac{1}{x}=\frac{26}{5}$ ?
A 3
B 5
C 7
D 9
E 11
29. Two similar cylinders, $A$ and $B$, have surface areas $350 \mathrm{~cm}^{2}$ and $5600 \mathrm{~cm}^{2}$, respectively. If the volume of cylinder A is $500 \mathrm{~cm}^{3}$, what is the volume of cylinder B in $\mathrm{cm}^{3}$ ?
A 8000
B 16,000
C 24,000
D 32,000
E 128,000
30. The first five terms of a sequence are $0,3,8,15,24, \ldots$ What is the 98th term?
A 9409
B 9506
C 9603
D 9605
E 9801
31. What is the factorised form of, $(x+y)^{2}+(x+y)$
A $(x+y+1)^{2}$
$\mathrm{B}(x+y)^{3}$
C $x^{2}+2 x y+y^{2}+x+y$
D $\left(x^{2}+y^{2}\right)(x+y)$
$\mathrm{E}(x+y)(x+y+1)$
32. 

$$
x^{3}-4 x^{2}-20 x+3 \div(x+3)
$$

A $x^{2}-7 x+1$
B $x^{2}+9 x-3$
C $x^{2}+7 x-1$
D $x^{2}-9 x+1$
E $x^{2}-9 x+3$
33.

Expand $\left(3 x^{2}-2 x+1\right)\left(4 x^{2}+3 x-7\right)$
A $12 x^{4}+3 x^{3}+27 x^{2}-17 x-7$
B $12 x^{4}-2 x^{3}+14 x^{2}+12 x-7$
C $12 x^{4}+x^{3}-27 x^{2}+19 x-7$
D $12 x^{4}+x^{3}-23 x^{2}+17 x-7$ E $12 x^{4}+2 x^{3}-23 x^{2}+17 x-7$
34. Which of the following is the equation of the straight line that passes through the points $(3,7)$ and $(1,5)$ ?
A $x+3 y=5$
B $x-y=-4$
C $3 x-2 y=4$
D $x+y=8$ E $x-y=-3$
35. What is the equation of the line which is perpendicular to the line $5 x-2 y=38$ and which passes through the point $(-3,14)$ ?
A $2 x+5 y=76$
B $5 x+2 y=64$
C $2 x-5 y=-76$
D $2 x+5 y=64$
E $5 x+2 y=13$
36. The equations of two lines are $x+y=14$, and $x-y=2$. What are the coordinates of the point of intersection?
A $(4,10)$
B (10, 4)
C $(7,7)$
D $(6,8)$
E (8, 6)
37. What is the equation of the line with gradient 2 and which passes through the point $(7,4)$ ?
A $2 x+y=4$
B $2 x-y=10$
C $2 x+y=10$
D $x-2 y=10$
E $x+2 y=3$
38. A square is drawn inside a semicircle as shown. If the radius of the semicircle is 1 , what is the area of the square?

A $\frac{4}{5}$
B $\frac{\sqrt{ } 3}{2}$
C $\frac{2}{3}$
D $\frac{\sqrt{ } 2}{3}$
E $\frac{\sqrt{ } 5+1}{2}$
39. The two roots of the quadratic equation, $x^{2}-85 x+k=0$ are both prime numbers. What is the sum of the digits of k ?
A 12
B 13
C 15
D 18
E 22
40. What is exact solution to this equation,

$$
\mathrm{x}^{x}=\frac{3}{4} \sqrt{ } 6
$$

A $\frac{3}{2}$
B $\frac{\sqrt{ } 2}{3}$
C $\frac{\sqrt{ } 3}{2}$
D $\frac{3 \sqrt{ } 3}{2 \sqrt{ } 2}$
E $\frac{2}{3}$
41. How many two-digit numbers can be written as the sum of six different powers of 2 , including $2^{0}$ ?
A 0
B 1
C 2
D 3
E 4
42. Which of the following is neither a square not a cube?
A $2^{9}$
B $3^{10}$
C $4^{11}$
D $5^{12}$
E $6^{13}$
43. Four straight lines intersect as shown.

What is the value of $a+b+c+d$ ?
A $180^{\circ}$
B $360^{\circ}$
C $540^{\circ}$
D $720^{\circ}$
E $900^{\circ}$

44. A nine-digit number is randomly composed using the digits from 1 to 9 . What is the probability that it is divisible by 18 ?
A $\frac{1}{3}$
B $\frac{4}{9}$
C $\frac{1}{2}$
D $\frac{3}{4}$
E $\frac{5}{9}$
45. If $5 x-y=18$ and $5 y-x=12$, what is the value of $x-y$ ?
A 0
B 1
C 3
D 6
E 9
46. If $a$ and $b$ are non-zero numbers, simplify,

$$
\left(a+\frac{1}{a}\right)^{2}+\left(b+\frac{1}{b}\right)^{2}+\left(a b+\frac{1}{a b}\right)^{2}-\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)\left(a b+\frac{1}{a b}\right)
$$

A 0
B 1
C 2
D 3
E 4
47. What is the maximum possible area of the rectangle shown?
A 28
B 45
C 48
D 49
E 120

48. Triangle $A B C$ is equilateral. The side $A B$ is the diameter of a circle of area $49 \pi \mathrm{~cm}^{2}$. $\mathrm{In} \mathrm{cm}^{2}$, what is the area of the triangle?

A $49 \sqrt{ } 3$
B $98 \sqrt{ } 2$
C $49 \sqrt{ } 2$
D $7 \sqrt{ } 3$
E $14 \sqrt{ } 3$
49. The figure contains a rectangle, with height 1 unit, a quadrant and a semicircle. Given that the two shaded parts have equal areas, find the length of PQ .

A $\sqrt{ } 2$
B $2 \sqrt{ } 2$
$\mathrm{C} 3 \sqrt{ } 2$
D $4 \sqrt{2}$
E $5 \sqrt{ } 2$
50. Following the pattern in the first three figures below, how many small triangles are there in the 7th figure?

A 244
B 294
C 352
D 414
E 480

## Answers

| 1. B | 11. D | 21. B | 31. E | 41. C |
| :--- | :---: | :---: | :---: | :---: |
| 2. C | 12. E | 22. E | 32. A | 42. E |
| 3. E | 13. B | 23. B | 33. D | 43. C |
| 4. C | 14. B | 24. C | 34. B | 44. |

## Answer Sheet

1. 





41.
42.

43.
44.
45.
46.
47.

48.
49.
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50.

41.
45.
46.

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