IVMO 2021 Senior
Time allowed - 1 Hour

1. By continual summing the digits of a number, you arrive at the digital root. What is the digital root of 23648157185?

A 1 B 3 C 5 D 7 E 9

2. The number, $4\overline{2}\overline{4}30\overline{7}\overline{3}$, has four vinculum digits. What is the correct number when the digits are expressed as positive integers?

A 3752927 B 3862937 C 4862037 D 4753126 E 3762927

3. In using Nikhilam multiplication, which of the following is correct?

A $\begin{array}{r} 752-258 \\ \times\, 997-003 \\ \hline 749 \quad 744 \end{array}$ B $\begin{array}{r} 752-357 \\ \times\, 997-002 \\ \hline 749 \quad 744 \end{array}$ C $\begin{array}{r} 752-248 \\ \times\, 997-003 \\ \hline 749 \quad 744 \end{array}$ D $\begin{array}{r} 752-246 \\ \times\, 997-004 \\ \hline 749 \quad 744 \end{array}$ E $\begin{array}{r} 752+248 \\ \times\, 997+003 \\ \hline 749 \quad 744 \end{array}$

4. What are the final three digits of 9999996^2 ?

A 992 B 016 C 486 D 996 E 136

5. When using Vertically and crosswise to calculate 2361×4215, what is the result of the fourth step before any carry digits are included?

A 29 B 16 C 31 D 32 E 23

6. When using Nikhilam multiplication, what are the vinculum digits for, 1023×991?

A $0\overline{2}\overline{3}$ B $00\overline{8}$ C $\overline{2}0\overline{7}$ D $\overline{2}\overline{1}\overline{9}$ E $\overline{7}0\overline{3}$

7. 12121212121×11

A 1333333331 B 13333333331 C 133333333331 D 1333333333331 E 13333333333331

8. The product of two prime numbers is 2021, which can be worked out using the rule, *When the final digits add to 10*. What are the two priime numbers?

A 79×61 B 83×17 C 37×23 D 53×67 E 43×47

9. Which fraction is the smallest?

A $\frac{1}{2000000008}$ B $\frac{2}{4000000015}$ C $\frac{4}{8000000036}$ D $\frac{8}{16000000056}$ E $\frac{2}{4000000017}$

10. What is the integer remainder when 1245003411 is divided by 999998?

A 9510 B 5901 C 1059 D 5190 E 1095

11. 1000004×1214623

A 1214627858492 B 1214628858492 C 1214627868492
D 1214627859492 E 1214627858482

12. What are the last four digits in the recurring pattern in the decimal equivalent of $\frac{1}{29}$?

A 7931 B 1369 C 2929 D 9631 E 0609

13. Which partial sum is correct when using a geometric sequence to calculate 31^3 ?

A $\begin{array}{r} 1\ 3\ 9\ 27 \\ \hline 6\ 18 \end{array}$ B $\begin{array}{r} 27\ 9\ 3\ 1 \\ \hline 27\ 9 \end{array}$ C $\begin{array}{r} 27\ 9\ 3\ 1 \\ \hline 9\ 27 \end{array}$ D $\begin{array}{r} 27\ 9\ 3\ 1 \\ \hline 18\ 6 \end{array}$ E None of these

14. What is the sum of the first twenty odd numbers?

A 200 B 300 C 400 D 500 E 600

15. The first five terms of a sequence are 0, 3, 8, 15, 24, ...
What is the 98th term?

A 9409 B 9506 C 9603 D 9605 E 9801

16. A nine-digit number is randomly composed using the digits from 1 to 9. What is the probability that it is divisible by 18?

A $\frac{1}{3}$ B $\frac{4}{9}$ C $\frac{1}{2}$ D $\frac{3}{4}$ E $\frac{5}{9}$

17. Which of the following is neither a square not a cube?

A 2^9 B 3^{10} C 4^{11} D 5^{12} E 6^{13}

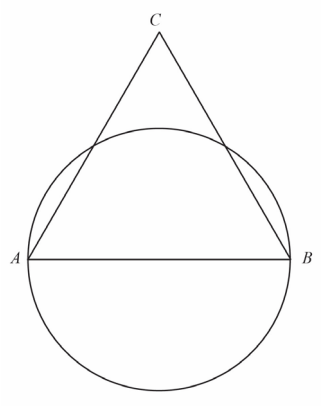
18. How many two-digit numbers can be written as the sum of six different powers of 2, including 2^0 ?

A 2 B 3 C 4 D 5 E 6

19. There are 100 students in a class. Exactly 99% of them have their books. How many students with books must leave the room to bring down the percentage with books to 98%?

A 1 B 2 C 10 D 49 E 50

20. Triangle ABC is equilateral. The side AB is the diameter of a circle of area 49π cm^2. In cm^2, what is the area of the triangle?



A $49\sqrt{3}$ B $98\sqrt{2}$ C $49\sqrt{2}$ D $7\sqrt{3}$ E $14\sqrt{3}$

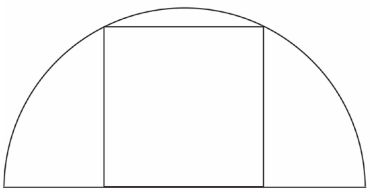
21. Pinocchio's nose is 4 cm long. Every time he tells a lie his nose doubles in length and every time he tells the truth his nose shortens by one third. He tells ten lies and one truth. Which of the following then corresponds most closely to the length of his nose?

A a cellphone B a bed C a tennis court D a football pitch E an airport runway

22. In 1960, a standard oil tanker was 130 metres long, 16 metres wide and 10 metres deep. It's volume, measured in deadweight tonnage, was 16,000 dwt. Ravi Tikoo then invented the supertanker with a length of 390 m, width 48 m and depth 30 m. What, approximately, was the deadweight tonnage of his supertanker?

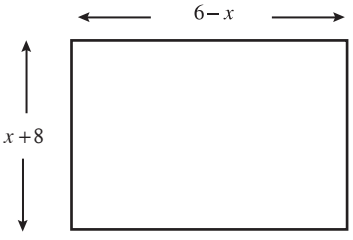
A 128,000 B 144,000 C 360,000 D 432,000 E 864,000

23. A square is drawn inside a semicircle as shown. If the radius of the semicircle is 1, what is the area of the square?



A $\frac{4}{5}$ B $\frac{\sqrt{3}}{2}$ C $\frac{2}{3}$ D $\frac{\sqrt{2}}{3}$ E $\frac{\sqrt{5}+1}{2}$

24. What is the maximum possible area of the rectangle shown?



A 28 B 45 C 48 D 49 E 120

25. What is the factorised form of, $(x+y)^2+(x+y)$

A $(x+y+1)^2$ B $(x+y)^3$ C $x^2+2xy+y^2+x+y$ D $(x^2+y^2)(x+y)$ E $(x+y)(x+y+1)$

26. Which of the following is the equation of the straight line that passes through the points (3, 7) and (1, 5)?

A $x+3y=5$ B $x-y=-4$ C $3x-2y=4$ D $x+y=8$ E $x-y=-3$

27. What is the equation of the line which is perpendicular to the line $5x-2y=38$ and which passes through the point $(-3,\ 14)$?

A $2x+5y=76$ B $5x+2y=64$ C $2x-5y=-76$ D $2x+5y=64$ E $5x+2y=13$

28. The equations of two lines are $x+y=14,$ and $x-y=2$. What are the coordinates of the point of intersection?

A (4, 10) B (10, 4) C (7, 7) D (6, 8) E (8, 6)

29. What is the equation of the line with gradient 2 and which passes through the point (7, 4)?

A $2x+y=4$ B $2x-y=10$ C $2x+y=10$ D $x-2y=10$ E $x+2y=3$

30. What is the radius of the circle with equation,

$$x^2+y^2-6x+14y+9=0?$$

A 9 B 28 C 6 D 3 E 7

31. If $5x-y=18$ and $5y-x=12$, what is the value of $x-y$?

A 0 B 1 C 3 D 6 E 9

32. Which of the following is a possible solution to the equation, $x+\frac{1}{x}=\frac{26}{5}$?

A 3 B 5 C 7 D 9 E 11

33. $(x - 1)$ is a factor of $x^4+x^3+x^2+x-k$. What is the value of k?

A 0 B 1 C 2 D 3 E 4

34. $x^3 - 4x^2 - 20x + 3 \div (x+3)$

A $x^2 - 7x + 1$ B $x^2 + 9x - 3$ C $x^2 + 7x - 1$ D $x^2 - 9x + 1$ E $x^2 - 9x + 3$

35. Expand $(3x^2 - 2x + 1)(4x^2 + 3x - 7)$

A $12x^4 + 3x^3 + 27x^2 - 17x - 7$ B $12x^4 - 2x^3 + 14x^2 + 12x - 7$ C $12x^4 + x^3 - 27x^2 + 19x - 7$

D $12x^4 + x^3 - 23x^2 + 17x - 7$ E $12x^4 + 2x^3 - 23x^2 + 17x - 7$

36. What is exact solution to this equation,

$$x^x = \frac{3}{4}\sqrt{6}$$

A $\frac{3}{2}$ B $\frac{\sqrt{2}}{3}$ C $\frac{\sqrt{3}}{2}$ D $\frac{3\sqrt{3}}{2\sqrt{2}}$ E $\frac{2}{3}$

37. The two roots of the quadratic equation, $x^2 - 85x + k = 0$ are both prime numbers. What is the sum of the digits of k?

A 12 B 13 C 15 D 18 E 22

38. For values of x between 0° and 360° inclusive, how many solutions are there to the equation,

$$2\cos x \sin 2x = \cos x$$

A 2 B 3 C 4 D 5 E 6

39. Which of the following is the derivative of $\sin(2x)e^{2x}$?

A $2e^{2x}(\sin 2x + \cos 2x)$ B $e^{2x}(\sin 2x + 2\cos 2x)$ C $e^{2x}(2\sin 2x + \cos 2x)$

D $4e^{2x}(\sin 2x + \cos 2x)$ E $2e^{2x}(\sin 2x - 2\cos 2x)$

40. Differentiate, $(2x^2 - 5x + 7)^5$

A $5(2x^2 - 5x + 7)^4$ B $5(4x - 5)(2x^2 - 5x + 7)^4$ C $10(2x^2 - 5x + 7)^4$

D $(4x - 5)(2x^2 - 5x + 7)^4$ E $4(4x - 5)(2x^2 - 5x + 7)^5$

41. Find the least value of k for which the line with equation, $y = 2x + k$, cuts the curve with equation, $y = x^2 - 10x + 21$.

A -8 B -10 C -15 D -17 E -21

42. Given that the following expression is equal to kn, what is the value of k?

$$(6n+1)^2 - (6n-1)^2$$

A 19 B 17 C 12 D 24 E 36

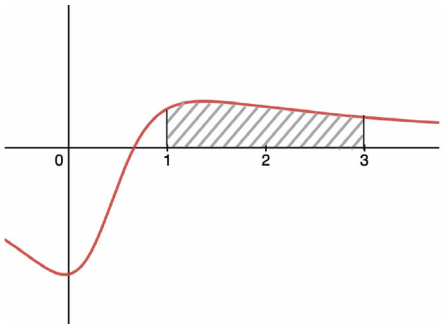
43. What are the first three terms in the series expansion for $\dfrac{2+3x-x^2}{(1+x)(2-3x)}$?

A $2+4x-x^2+\cdots$ B $1+3x-2x^2+\cdots$ C $2+3x-4x^2+\cdots$

D $1+2x+2x^2+\cdots$ E $1-2x-6x^2+\cdots$

44. The curve shown has equation, $\dfrac{3x-2}{3x^2-4x+3}$.

Which of the following gives the area of the shaded region expressed as $a\ln b$ where a and b are rational?



A $\frac{1}{2}\ln 3$ B $4\ln 18$ C $\frac{1}{4}\ln\frac{9}{2}$ D $2\ln\frac{9}{2}$ E $\ln 3$

45. Given two triples, A) 3 4 5 and B) 24 7 25, what is the triple for A + B?

A 44 75 125 B 44 117 125 C 75 44 125

D −75 −44 125 E 156 65 125

46. Angle A is given by the triple, A) 3 4 5. Which of the following is a triple for $\frac{1}{2}A$?

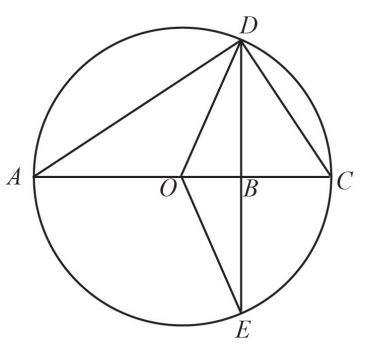
A 1 2 √5 B 2 1 √5 C 5 5 5√2 D 4 3 5 E 8 5 √89

47. If a and b are non-zero numbers, simplify,

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(ab+\frac{1}{ab}\right)^2-\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)\left(ab+\frac{1}{ab}\right)$$

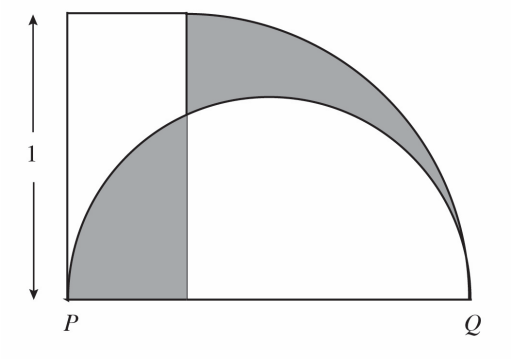
A 0 B 1 C 2 D 3 E 4

48. In the figure, A, E, C, D are points on the circumference of a circle centre O. AC is a diameter and ED is perpendicular to AC. Which line has a length equal to the geometric mean of AB and BC?



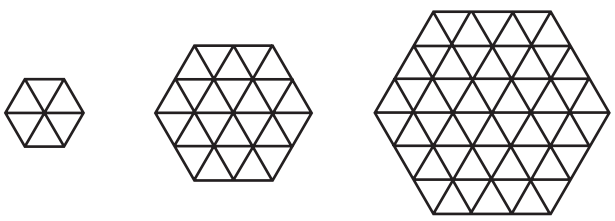
A AD B DC C BD D DE E OD

49. The figure contains a rectangle, with height 1 unit, a quadrant and a semicircle. Given that the two shaded parts have equal areas, find the length of PQ.



A $\sqrt{2}$ B $2\sqrt{2}$ C $3\sqrt{2}$ D $4\sqrt{2}$ E $5\sqrt{2}$

50. Following the pattern in the first three figures below, how many small triangles are there in the 7th figure?



A 244 B 294 C 352 D 414 E 480

Answers - Senior IVMO 2021

1. C	**11.** A	**21.** C	**31.** B	**41.** C
2. E	**12.** A	**22.** D	**32.** B	**42.** D
3. C	**13.** D	**23.** A	**33.** E	**43.** D
4. B	**14.** C	**24.** D	**34.** A	**44.** E
5. A	**15.** C	**25.** E	**35.** D	**45.** B
6. C	**16.** B	**26.** B	**36.** A	**46.** B
7. C	**17.** E	**27.** D	**37.** B	**47.** E
8. E	**18.** A	**28.** E	**38.** E	**48.** C
9. C	**19.** E	**29.** B	**39.** A	**49.** A
10. B	**20.** A	**30.** E	**40.** B	**50.** B

Answer Sheet

1.	11.	21.	31.	41.
2.	12.	22.	32.	42.
3.	13.	23.	33.	43.
4.	14.	24.	34.	44.
5.	15.	25.	35.	45.
6.	16.	26.	36.	46.
7.	17.	27.	37.	47.
8.	18.	28.	38.	48.
9.	19.	29.	39.	49.
10.	20.	30.	40.	50.